State of the art and present developments of a general approach for GPS-based height determination

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ABSTRACT

The contribution treats a sophisticated concept in the area of GPS-based height determination with components being appropriate to branch out into different classes of standard approaches, depending on the kind of data sources as well as on the principal target. So, besides a GPS-based height determination, also a height system transformation may be set up. Basically any kind of height data, namely geoid models N (e.g. EGG97), heights H, levelling ΔH, GPS heights h and GPS baselines Δh may be combined. Partly a Finite Element Model (FEM) is set up for the representation of height reference surfaces (HRS). This FEM is parametrized by bivariate polynomials sets, and continuity conditions guarantee a continuous transition of the FEM surface along the edges of neighbouring meshes in any area size. In opposite to digital terrain models, the nodes of the FEM mesh may differ from the position of the data used for the FEM determination.

The first part of the contribution treats the class of already practical working standard approaches, developed to transform in a statistically controlled way ellipsoidal GPS heights h into heights H of a standard height system. First the role of use and the kind of a datum and systematics adaption of geoid models N in a GPS height integration are discussed. The „geoid refinement approach“ standard means that a datum adapted geoid model N is used as direct observation, while the FEM serves as additional overlay to improve the final representation of the HRS. The special case of the „pure FEM approach“ arises, if the FEM representation of HRS is computed purely by geometric observations H, ΔH, h, Δh. The „pure geoid approach“ means, that only a datum adapted geoid model N is used in a GPS height integration. The three approaches provide a flexible area of models implemented in the software HEIDI2. Different pilot projects in several parts of Europe finished successfully, and the height integration concept is meanwhile used as a standard in some state survey agencies. The experience shows that a high precision level for a GPS based height determination up to a 5 mm level in rather large areas is achieved, e.g. using the EGG97.

The second part and class of approaches treats the application of the FEM component for the purpose of height system transformation (e.g. conversion of NN-heights to normal heights).

The third part of the presentation and class of approaches considers the so called general approach, where the HRS is completely established by a FEM, using different datum adapted geoid models N, terrestrial height information H and ellipsoidal GPS heights h as data sources. The result of the computation and "geoid mapping" respectively, leads to a Digital FEM Height Reference Surface (DFHRS). The DFHRS may be set up as data base for a datum free direct GPS-based online heighting in DGPS networks. First results of a pilot project in the German SAPOS network are reported.

1. INTRODUCTION

The transformation of a geocentric cartesian position (x,y,z) determined from DGPS provides the plane position represented by the geographical latitude and longitude (B,L) and the ellipsoidal height h, all referring to the datum of the respective reference station(s) used in DGPS. Both (B,L) and h depend on the metric and shape of the reference ellipsoid (a=main axis, f=flattening) used in the computation of (B,L,h). In general the GRS80 or the WGS84 ellipsoid are used in the context with GPS. The transition of GPS results $(B,L,h)_1$, in the following described as system 1 to a set of national network coordinates $(B,L,h)_2$, described as system 2, is to be performed by a threedimensional similarity transformation. There three translations (u,v,w), three rotations ($\varepsilon_x,\varepsilon_y,\varepsilon_z$) and one scale difference Δm between both datum systems have to be taken into account. Using a taylor series expansion with linearization point $(B,L,h)_1$ and assuming small rotations we may write the datum transition from system 1 to system 2 on splitting the threedimensional problem equivalently into the plane and the height component in the following way [9],[5]:

Plane components (1), (2) of the threedimensional datum transition

$$
\begin{aligned}
B_2 &= B_1 + \partial B_1(d) = B_1 + \partial B_1(u,v,w,\varepsilon_x,\varepsilon_y,\varepsilon_z,\Delta m,\Delta a,\Delta f) \\
&= B_1 + \left[\frac{-\cos(L)\cdot\sin(B)}{M+h}\right]_1 \cdot u + \left[\frac{-\sin(L)\cdot\sin(B)}{M+h}\right]_1 \cdot v + \left[\frac{\cos(B)}{M+h}\right]_1 \cdot w + \\
&\left[\sin(L)\cdot\frac{h+N\cdot W^2}{M+h}\right]_1 \cdot \varepsilon_x + \left[-\cos(L)\frac{h+N\cdot W^2}{M+h}\right]_1 \cdot \varepsilon_y + [0]\cdot\varepsilon_z + \\
&\left[\frac{-e^2\cdot N\cdot\cos(B)\cdot\sin(B)}{M+h}\right]_1 \cdot \Delta m + \left[\frac{N\cdot e^2\cdot\cos(B)\cdot\sin(B)}{a\cdot(M+h)}\right]_1 \cdot \Delta a + \left[\frac{M\cdot\sin(B)\cdot\cos(B)\cdot(W^2+1)}{(M+h)\cdot(1-f)}\right]_1 \cdot \Delta f
\end{aligned} \quad (1)
$$

$$
\begin{aligned}
L_2 &= L_1 + \partial L_1(d) = L_1 + \partial L_1(u,v,w,\varepsilon_x,\varepsilon_y,\varepsilon_z,\Delta m,\Delta a,\Delta f) \\
&= L_1 + \left[\frac{-\sin(L)}{(N+h)\cdot\cos(B)}\right]_1 \cdot u + \left[\frac{\cos(L)}{(N+h)\cdot\cos(B)}\right]_1 \cdot v + [0]\cdot w + \\
&\left[\frac{-(h+(1-e^2)\cdot N}{(N+h)\cdot\cos(B)}\cdot\cos(L)\cdot\sin(B)\right]_1 \cdot \varepsilon_x + \left[\frac{-(h+(1-e^2)\cdot N)}{(N+h)\cdot\cos(B)}\cdot\sin(L)\cdot\sin(B)\right]_1 \cdot \varepsilon_y + [1]\cdot\varepsilon_z + \\
&[0]\cdot\Delta m + [0]\cdot\Delta a + [0]\cdot\Delta f
\end{aligned} \quad (2)
$$

Ellipsoidal height component (3) of a threedimensional datum transition

$$
\begin{aligned}
h_2 &= h_1 + \partial h_1(d) = h_1 + \partial h_1(u,v,w,\varepsilon_x,\varepsilon_y,\varepsilon_z,\Delta m,\Delta a,\Delta f) \\
&= h_1 + [\cos(L)\cdot\cos(B)]_1 \cdot u + [\cos(B)\cdot\sin(L)]_1 \cdot v + [\sin(B)]_1 \cdot w + \\
&[e^2\cdot N\cdot\sin(B)\cdot\cos(B)\cdot\sin(L)]_1 \cdot \varepsilon_x + [-e^2\cdot N\cdot\sin(B)\cdot\cos(B)\cdot\cos(L)]_1 \cdot \varepsilon_y + [0]\cdot\varepsilon_z + \\
&[h+W^2\cdot N]_1 \cdot \Delta m + \left[\frac{-N\cdot W^2}{a}\right]_1 \cdot \Delta a + \left[\frac{W^2\cdot M\cdot\sin^2(B)}{1-f}\right]_1 \cdot \Delta f
\end{aligned} \quad (3)
$$

N(B) and M(B) are introduced as the latitude dependent quantities of the so called normal and the meridian radius of curvature respectively. For W(B) and e^2 we have W(B)=a/N(B) and $e^2=2f-f^2$. In general the parameter changes Δa and Δf are known, and the respective quantities are introduced as deterministic corrections. Respective corrections due to Δa and Δf are therfore not mentioned in the following.

The integration of GPS-results may be carried out with respect to (1), (2) by using only plane coordinates $(B,L)_2$ in the domain of the identical points with respect to the determination of the datum parameter set $\mathbf{d}=(u,v,w,\varepsilon_x,\varepsilon_y,\varepsilon_z,\Delta m)$. This has the advantage, that in addition to the ellipsoidal GPS height h_1 no further height information (ellipsoidal height h_2, geoid N_G and standard heights H_2) is necessary from the national network system 2 [14].

If we take vice versa a look to (3) we need in the context with $H_2=h_2-N_G$ the introduction of a so called „geoid“ model N_G, as we generally dispose only on the standard heights H_2 referring to the physically defined height reference system HRS (fig.1). But in practice we have to consider, that a geoid model N_G taken from a geoid data base [3] has – being another surface in space – an own more or less small but unknwon datum $\mathbf{d}_G=(u,v,w,\varepsilon_x,\varepsilon_y,\Delta m)_G$.

Fig. 1: Ellipsoidal GPS height h, height reference surface HRS or briefly „geoid“ and earth surface EOF at a point P(B,L)

Instead of the „raw“ ideal and datum effects neglecting standard formula

$$H = h - N_G \tag{4a}$$

we arrive starting from (3) in real life practice at the complete formula reading

$$H_2 = h_1 + \partial h_1(u,v,w,e_x,e_y,??m) - (N_G + \partial N(u,v,w,e_x,e_y,??m_G)) = h_1 + \partial h_1(d) - (N_G + \partial N(d_G)) \; . \tag{4b}$$

From (1), (2) and (4b) follows that a threedimensional GPS-integration based on standard heights H_2 and a geoid model N_G has to consider in total 13 parameters within **d** and $\mathbf{d}_G$. Assuming that the data base geoid values N_G are referred to $(B,L)_1$, we see directly that the parameters within the different sets **d** and $\mathbf{d}_G$ separate due to the variation of the heights h_1 and N_G within the coefficients belonging to **d** and $\mathbf{d}_G$ respectively (1),(2),(3). The standard approach of a threedimensional transformation with only one common set of seven parameters **d** is therefore not free of systematic errors.

If we however restrict a GPS-integration to the isolated GPS height integration problem, meaning the transformation of GPS heights h_1 to standard heights H_2, we derive from (4b) :

$$\begin{aligned} H_2 = h_1 - N_G &+ [\cos(L)\cdot\cos(B)]_1 (u - u_G) + [\cos(B)\cdot\sin(L)]_1 \cdot (v - v_G) + [\sin(B)]_1 \cdot (w - w_G) + \\ &[e^2 \cdot N \cdot \sin(B)\cdot\cos(B)\cdot\sin(L)]_1 \cdot (\varepsilon_x - \varepsilon_{x,G}) + [-e^2 \cdot N \cdot \sin(B)\cdot\cos(B)\cdot\cos(L)]_1 \cdot (\varepsilon_y - \varepsilon_{y,G}) + \\ &[h_1 + W^2 \cdot N]\cdot\Delta m - [N_G + W^2 \cdot N]\cdot\Delta m_G \\ = h_1 - N_G &+ \partial_{h_1,G}(u',v',w',\varepsilon_x',\varepsilon_y') + [h_1 + W^2 \cdot N]\cdot\Delta m - [N_G + W^2 \cdot N]\cdot\Delta m_G \end{aligned} \tag{5}$$

We recognize from (5), that due to some common coefficients one set of parameter-differences $\mathbf{d}'=(u'=u-u_G;\ v'=v-v_G;\ w'=w-w_G;\ \varepsilon_x'=\varepsilon_x-\varepsilon_{x,G};\ \varepsilon_y'=\varepsilon_y-\varepsilon_{y,G})$ may be introduced instead of two different sets in a geoid-model based GPS-height integration (8a,b,c). Separate parameters have to be kept only for the scale parameters Δm and Δm_G .

2. STANDARDS OF GPS HEIGHT INTEGRATION

With the trend of replacing old national datum systems in favour of ITRF-related datum systems and respective DGPS reference station systems (like e.g. SAPOS in Germany), the datum problem for the plane component (B,L) in GPS-based positioning will vanish by and by. But for the reason of a physically different height reference surface HRS for the standard heights H (fig. 1) defined by geopotential numbers, the problem of a transition of ellipsoidal

GPS-heights h_1 to the standard heights H_2 referring to a HRS – or briefly spoken „geoid“ (a true geoid for an orthometric height system, a quasi-geoid for a normal height system etc.) – will remain .

Different approaches have been developed from the „Karlsruhe working group“ [7] up to now. The advantages of the above splitting into the plane (1),(2) and height component (5) led to a powerful and flexible set of GPS-integration approaches [4], [5], [6], [7] which will be presented in the following chapters.

2.1 Finite Element Representation (NFEM) of the Height Reference Surface (HRS)

A powerful and central tool within the GPS height integration approaches of the „Karslruhe working group“ consists in the representation of the „geoid“ N_G or better the height reference surface HRS (fig. 1) as a finite element surface. In this way the finite element model NFEM(**p,**B,L) of HRS represents in the ideal sense $h=H+N_G$ - datum free and independent of the type of the standard height system H - the height N_G of the HRS over the ellipsoid as a function of the plane position (B,L) and the parameter vector **p**. As described in details in [4] and [5] the finite element representation NFEM(**p**,y,x) of the HRS is performed over a square grid with nodal points. The plane position (B,L) is replaced by metric coordinates (y(B,L)=“Eastern“ and x(B,L)=“Northern“) such as UTM or Gauß-Krüger coordinates to be computed from (B,L).

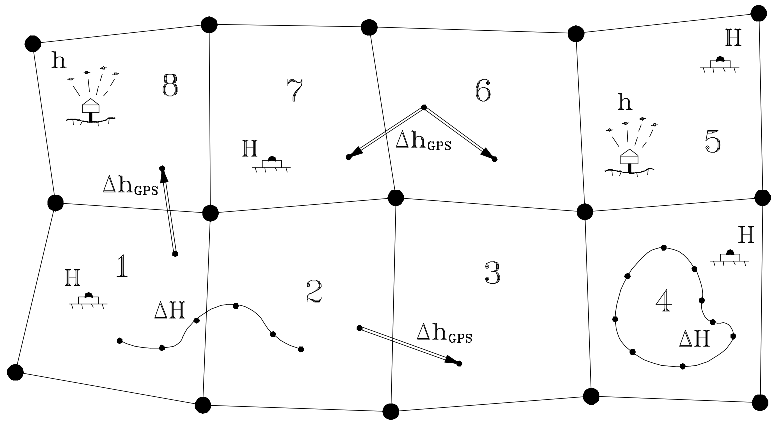

Fig. 2: Nodal points • and edges of a FEM-meshing and geodetic measurements (h,H,ΔH,Δh) used (eventually together with a geoid-model N_G as additional data source) for the determination of the NFEM(**p**,y,x) model of the HRS.

The mesh size and shape (fig.2) and at the same time the approximation quality of NFEM(**p**,y,x) with respect to the true HRS (fig. 1) may be chosen arbitrary. A special advantage and characteristic of the NFEM(**p**,y,x) representation consists last but not least in the fact, that the nodal points (•, fig. 2) of the FEM grid are totally independent of the geodetic network and data points (h,H,ΔH,Δh and geoid data N_G), which are used for the determination of the parameter vector **p** of NFEM(**p**,y,x). Without loss of generality we choose in the following bivariate polynomials of degree l as basic function to carry the surface NFEM(**p**,y,x) within the different meshes. The corresponding polynomial coefficients are introduced as $a_{ij,k}$, so that the parameter vector **p** consists of all coefficiental sets $a_{ij,k}$ over m meshes (i=0,l; j=0,l and k=1,m).

$$\mathrm{NFEM}(p) =: \begin{cases} N(p_k) = \sum_{i=0}^{l} \sum_{j=0}^{l-i} a_{ij,k} \cdot y^i \cdot x^j \\ C_{0,1,2}(p_m, p_n) \end{cases} \qquad (6)$$

Dependent on the plane position (y,x) the local „geoid height" N_G is to be received from the finite element representation NFEM(**p**,y,x) by first identifying the corresponding k-th mesh according to the position (y,x) by means of the vector of nodal point positions. Then N_G is to be evaluated from NFEM(**p**,y,x) by the local polynomial with coefficients $a_{ij,k}$ at the plane position (y,x).

To imply a continuous surface NFEM(**p**,y,x) one set of continuity conditions of different type $C_{0,1,2}$ has to be set up at the computation of NFEM(**p,**y,x) for each couple of neighbouring meshes m and n. The continuity type C_0 implies the same functional values along each common mesh border. The continuity type C_1 implies the same tangential planes and the continuity type C_2 the same curvature along the common borders of the HRS model NFEM(**p**,y,x). The continuity conditions occur as additional condition equations related to the polynomial sets of the coefficients $a_{ij,m}$ and $a_{ij,n}$ of each couple of neighbouring meshes m and n. The number and the mathematical contents of these condition equations depend on the polynomial degree l as well as on the continuity equation type [5].

The standard in the application of NFEM(**p**,y,x) in GPS height integration research and projects up to now was to use C_0 conditions and a degree of l=1 for a small mesh sizes up to 10 km, and degrees l=2,3 for larger mesh sizes. For the case l=3 and C_o-continuity for NFEM(**p**,y,x) we have to introduce for each neighbouring mesh border the following condition equations [5] :

$$da_{30}\, dx^3 + da_{21}\, dx^2 dy + da_{12}\, dxdy^2 + da_{03} dy^3 = 0 \qquad (7a)$$
$$da_{30}\Delta^3 + da_{20}\Delta^2 dy + da_{10}\Delta dy^2 + da_{00} dy^3 = 0 \qquad (7b)$$
$$da_{10} dxdy^2 + da_{01} dy^3 + 2da_{20}\Delta dxdy + da_{11}\Delta dy^2 + 3da_{30}\Delta^2 dx + da_{12}\Delta^2 dy = 0 \qquad (7c)$$
$$da_{20} dx^2 dy + da_{11} dxdy^2 + da_{02} dy^3 + 3da_{30}\Delta dx^2 + 2da_{21}\Delta dxdy + da_{12}\Delta dy^2 = 0 \qquad (7d)$$

With respect to the known nodal points $A(y_A,x_A)$ and $E(y_E,x_E)$ of the mesh grid (fig. 2) we introduced the abbreviations $dx=x_E-x_A$ and $dy=y_E-y_A$ as well as $\Delta=dy\cdot x_E-dx\cdot y_A$ and $da_{ij}=a_{ij,m}-a_{ij,n}$ in (7a,b,c,d).

2.2 Standard approaches of GPS height integration

Starting with formula (5) we immediately arrive at the so called „pure geoid approach". This approach is to be applied in a GPS height integration, as soon as good geoid information $N_G(B,L)$ is available. The parameters fo the datum part $\partial_{h,G}(\mathbf{d}',\Delta m_G)$ have to be estimated. With some simplification in the scale term[1] for Δm, the „pure geoid approach" reads in the corresponding system of observation equations as follows:

$$h + v = m\cdot H + N_G \qquad (8a)$$
$$N_G(B,L) + v = N_G + \partial_{h,G}(\mathbf{d}', \Delta m_G) \qquad (8b)$$
$$H + v = H \qquad (8c)$$

[1] The scale term following (3) looks like the expression for m in (12a).

GPS heights h, a geoid model N_G (B,L) and terrestrial heights H may be used as obervations. Of course the formulas are easy to extend to levelling ΔH and GPS height baselines Δh, which are also both included in all subsequent approaches. Apart from the datum part $\partial_{h,G}$ ($\mathbf{d}'$,Δm_G) the geoid model N_G (B,L) is treated as so called ‚direct observation‘. The datum part $\partial_{h,G}$ ($\mathbf{d}'$,Δm_G) may also model and remove some systematics [4], [5], [6], [7].

In polarity to (8a,b,c) and above all in the case, that no geoid-information exists, we may derive the HRS completely from the observations (h,H,ΔH,Δh) as finite element representation NFEM($\mathbf{p}$,y,x) of the HRS as given in (6). This approach is called the „pure finite element approach". It reads:

$$h + v = m\cdot H + NFEM(\mathbf{p},y,x) \quad (9a)$$
$$H + v = H \quad (9b)$$

The powerful synergy of both above approaches finally leads to the so called „geoid-refinement approach". It is used for the case that the available geoid information N_G (B,L) is to be refined by a finite element model NFEM($\mathbf{p}$,y,x), which is acting as additional overlay to improve the geoid model (fig. 3). The geoid-refinement approach reads:

$$h + v = m\cdot H + N_G \quad (10a)$$
$$N_G(B,L) + v = N_G + \partial_{h,G}(\mathbf{d}', \Delta m_G) + NFEM(\mathbf{p},y,x) \quad (10b)$$
$$H + v = H \quad (10c)$$

All above GPS-height integration approaches are implemented as standard models in the software package HEIDI2 ©Dinter/Illner/Jäger. The approaches are described in different papers and were proved in different projects [2],[4],[5],[6],[7],[8],[10],[11],[12].

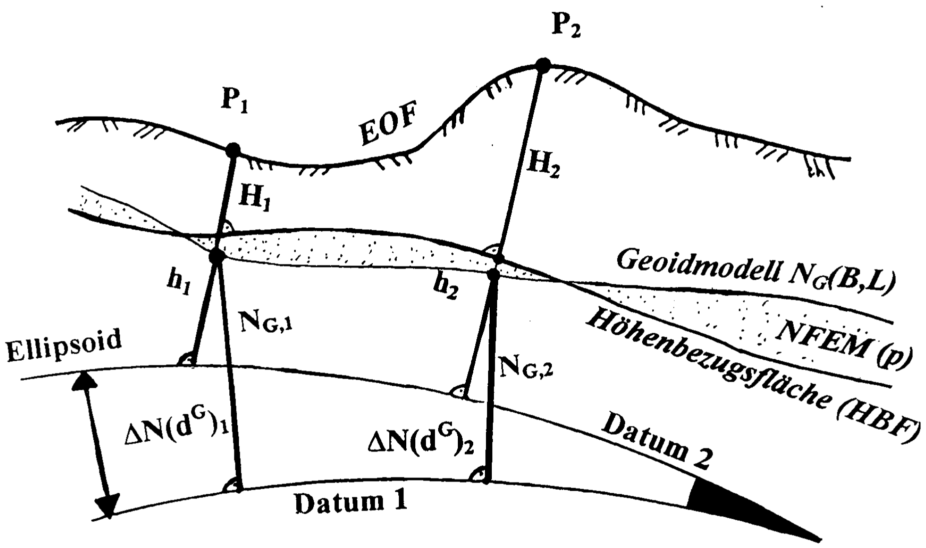

Fig. 3: Geoid-refinement approach as a synergetic combination of geoid information N_G(B,L) submitted to a datum change (datum 1 -> datum 2) and the finite element model NFEM($\mathbf{p}$,y,x). NFEM($\mathbf{p}$,y,x) is introduced to model remaining systematics (dotted) between the introduced geoid model N_G(B,L) and the true height reference surface HRS (=" Höhenbezugsfläche (HBF)").

2.3 Example of a GPS height integration performed with the software HEIDI2

The following example of a GPS height integration treats the use of the commercially available EGG97 geoid model [3] for an integration of GPS heights h into the normal height system H of the height network of Tallinn, Estonia. The network has an extension of 40 km by 25 km. The computations were done by the author in the frame of a TEMPUS project between the Tallinn Technical University, the University of Technology Karlsruhe and other European universities. The given 23 ellipsoidal GPS-heights h in the EST92 datum were introduced with a quality of 3 mm, as proved before in a free adjustment of the respective GPS height baselines. The given normal heights were introduced with a quality of 3mm, and the EGG97 observations $N_G(B,L)$ with a precision of 5 mm. The different versions of the GPS height integration were computed on the base of the pure geoid approach (8a,b,c) with the software HEIDI2.

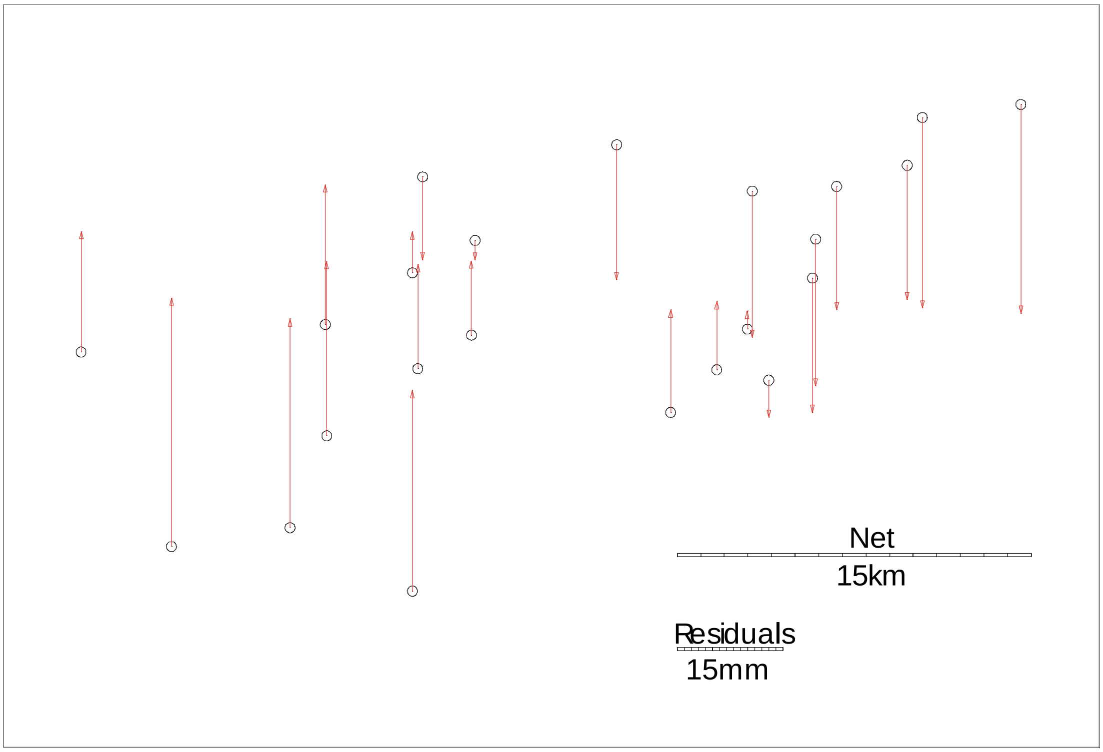

Fig. 4: GPS height integration for the Tallinn network by the pure geoid approach without taking a necessary datum-transition $\partial_{h,G}$ (**d'**,Δm_G) part for h and N_G into account: The residuals in the known control points - treated as new points - show the systematics of a datum tilt up to ± 3.5 cm.

The result of a first version, where - in sense of the unrealistic ideal (4a) - no datum transition $\partial_{h,G}$(**d'**,Δm_G) for h and N_G was introduced, is presented in fig. 4. Each known point was once computed as a „new point“ determined by „GPS and geoid“. The residuals in the identical points H are in the range of up to ± 3.5 cm and show the typical effect of a neglected datum tilt in this high range.

The fig. 5 shows the next set of computations in the pure geoid approach (8a,b,c) used as computation model for a GPS height integration. Now a datum transition $\partial_{h,G}$(**d'**,Δm_G) was taken into account. The residuals in the known control remain less than 1 cm, the mean residual is in the range of ± 4 mm. In this version of a GPS-based height integration all observation components are consistent with their assumed a priori precision and no gross errors occur in all runs. An additional geoid refinement might be computed by the geoid refinement approach (10a,b,c).

For further examples of GPS height integration in medium and in large scale networks and also due to the other above approaches like the geoid refinement approach (10a,b,c) and the pure FEM approach (9a,b) it is referred to [4],[5],[6],[7],[10],[11],[12].

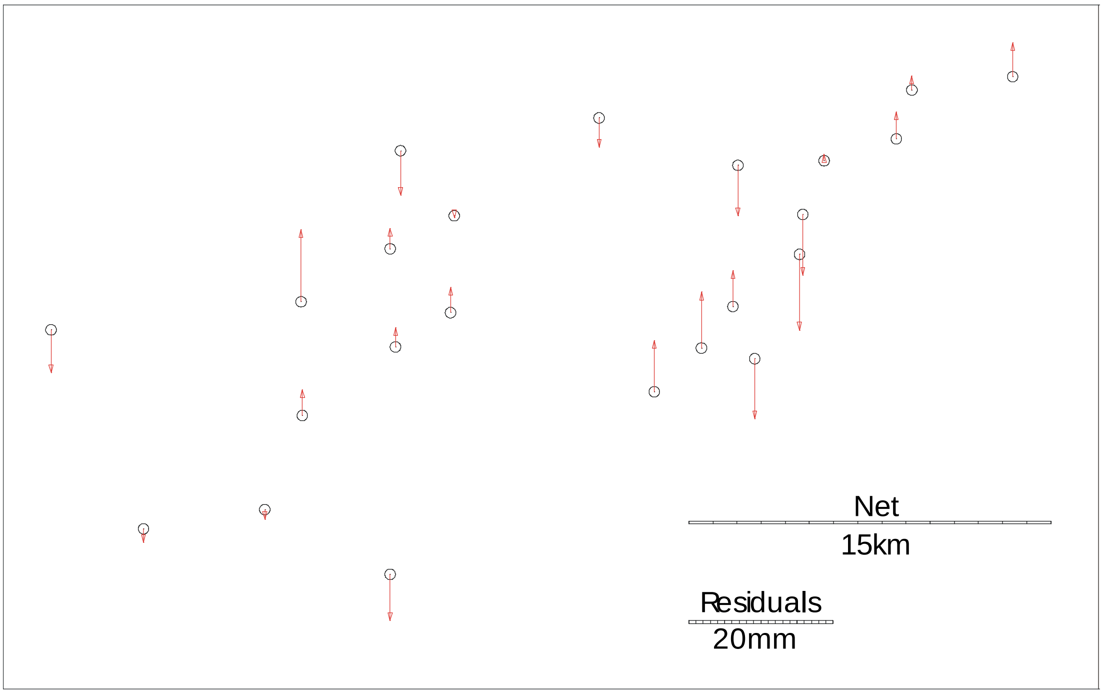

Fig. 5: GPS height integration for the Tallinn network by the pure geoid approach on taking the necessary datum-transition $\partial_{h,G}$ ($\mathbf{d}$',Δm_G) part for h and N_G into account: The residuals in the known control points - treated as new points – keep in the mean range of only ± 4mm.

3. HEIGHT SYSTEM TRANSFORMATION

The essential components of the above GPS height integration concept - namely the datum transformation part for heights (3) and the finite element representation NFEM($\mathbf{p}$,y,x) of a HRS (6) - may be transferred to the problem of transforming old heights H_{old} to new heights H_{new} of a new height system. In analogy to the above „geoid refinement approach“ the most general approach for a height system transformation reads:

$$H_{old} + v = H_{new} + \partial H(\mathbf{d}) + NFEM(\mathbf{p},y,x) \quad (11a)$$
$$H_{new} + v = H_{new} \quad (11b)$$

The datum transformation parameters **d** as well as the parameters **p** of the finite element model are to be determined by identical points (H_{old} ,H_{new}) in both systems.

4. ONLINE GPS-HEIGHTING – PRODUCTION AND APPLICATION OF A DIGITAL FINITE ELEMENT HEIGHT REFERENCE SURFACE

4.1 Digital Finite Element Height Reference Surface (DFHRS) concept for an online GPS-Heighting

The profile and target of an online height positioning is easy to formulate (see fig. 6): An ellipsoidal GPS-height h, determined at a position (y,x), has to be converted directly to the height H of the standard height system. The converted height H should result online after applying a correction to h, and the resulting H should not suffer with a quality-decrease compared to the heights H resulting from a GPS height integration in postprocessing (approaches chap. 2)

In this chapter a general concept is presented, which fulfils all above requests and shows besides this even some more positive aspects. The concept is to produce in a first step in a controlled way a so called **D**igital-**F**inite-**E**lement-**H**eight-**R**eference-**S**urface (DFHRS) as a new kind of data base product (= production step). The second step is to make this data base accessible online – in an active or passive way - for DGPS heighting (= application step). That means, that either the DGPS user has the DFHRS at his disposal or the DGPS service exclusively uses the DFHRS for the evaluation of a correction Δ to transform a GPS height h to the height H of the standard height system (principle, see fig. 6).

The production step of the DFHRS reads in the system of observation equations as follows:

$$h + v = H - (h+N\cdot W^2)\cdot \Delta m + NFEM(\mathbf{p},x,y) \qquad (12a)$$
$$N_G(B,L) + v = NFEM(\mathbf{p},y,x) - \partial_{h,G}(\mathbf{d'}, \Delta m_G) \qquad (12b)$$
$$H + v = H \qquad (12c)$$

Identical points (H, h) and if available, one or a number of geoid models $N_G(B,L)$ are used as observations to produce the DFHRS. The DFHRS on the right side is represented completely by the finite element model NFEM(**p**,x,y) of the HRS. NFEM(**p**,y,x) is modeled like in (6) with continuity conditions. The geoid model input N_G (B,L) is „mapped“ to the DFHRS by removing the datum part $\partial_{h,G}(\mathbf{d'}, \Delta m_G)$. An additional NFEM-refinement term may be set up in (12b). The production step of the DFHRS (12a,b,c) has to be embedded in a statistical quality control concept, e.g. of a least squares estimation, so that any component including the input of „mapped“ and datum-adapted geoid-model, can be controlled.

The decisive components of the production step, which are afterwards needed in the application step - namely in an online GPS-heighting - are contained in (12a). Equation (12a) leads to the following correction scheme, which has to be applied to the GPS height h in an online application of the DFHRS data base:

$$H = h + \Delta = h + corr1 + corr2 = h - NFEM(\mathbf{p},y,x) + (h+N\cdot W^2)\cdot \Delta m \qquad (13)$$

The first correction part „corr1“ is due to the DFHRS („geoid correction“), and „corr2“ is due to the scale Δm between the GPS heights h and those of the standard height system H.

4.2 Example of DFHRS production

Fig. 7 shows the finite element grid of the 30km by 30km „Mosbach“ area near Heidelberg, where a DFHRS was computed in the frame of a pilot project [1]. A first kind of DFHRS was produced using only identical points (H, h) in both systems, without geoid information $N_G(B,L)$ and a respective „geoid mapping“ (12b). The DFHRS was evaluated for this case with a polynomial degree l=1 over a 16 mesh grid. In addition to the scale parameter Δm the complete DFHRS for the area (fig. 7) could in this case be represented by k=16 sets of each three coefficients $\mathbf{p}_k=(a_{oo},a_{o1},a_{1o})_k$. For the special case that besides the identical points (h,H, see fig 7.) no geoid information was used for the evaluation of the DFHRS, the precision of the $\mathbf{p}_k$ of the DFHRS restricts the DGPS-based online heighting to a quality range of (1-3) cm.

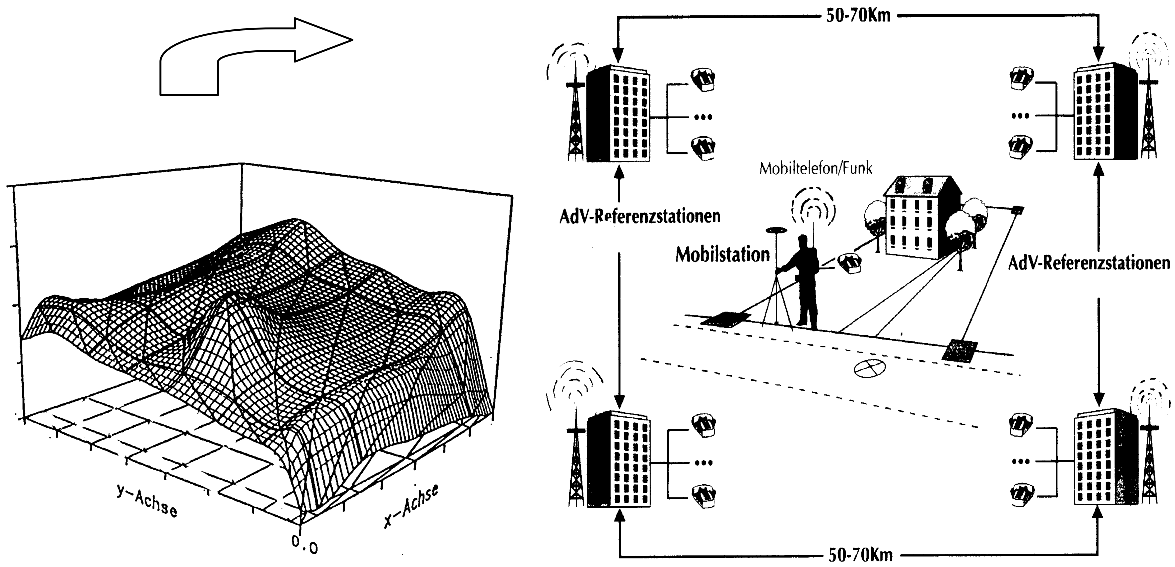

Fig 6: DFHRS (left) and its use (right) as DFHRS data base for a DGPS-based online heighting (h $\rightarrow$ H).

A quality increase and simultaneously a reduction of the number of identical points is achieved by an additional „geoid mapping“ (12b). The resulting DFHRS (12a,b,c,d) is then better than the geoid input, meaning that the additional mapping of e.g. the EGG97 cm-geoid will provide a cm or sub-cm quality for the resulting DFHRS product, as proved in respective investigations [12].

4.3 Outlook for the DFHRS concept

The DFHRS can be characterized as a new product appropriate for an online GPS-heighting with best quality and economical properties. The wellknown datum problem and individual datum calibration steps using identical points (h,H) in/before GPS heighting are completely dropped out. The DFHRS enables a direct GPS-heighting with a general usability for anybody in the frame of DGPS-applications and DGPS services.

The production of the DFHRS (12a,b,c) is performed as an overdetermined least squares adjustment, which enables a quality control of all components including the input of geoid models. The computation of the DFHRS product may be repeated at any time, as soon as new data (h, H, ΔH, Δh and N_G) arise, or even if a change of the height system type or datum is in-tended.

The variation of the mesh size further enables to produce on demand different DFHRS products with a different geometric quality (and prize) for an online height positioning purpose. Besides that there are two different ways for a DFHRS marketing: The first way is to keep the DFHRS on the side of the data- and DFHRS owner and transmit only the correction Δ (13) by the DGPS-service.

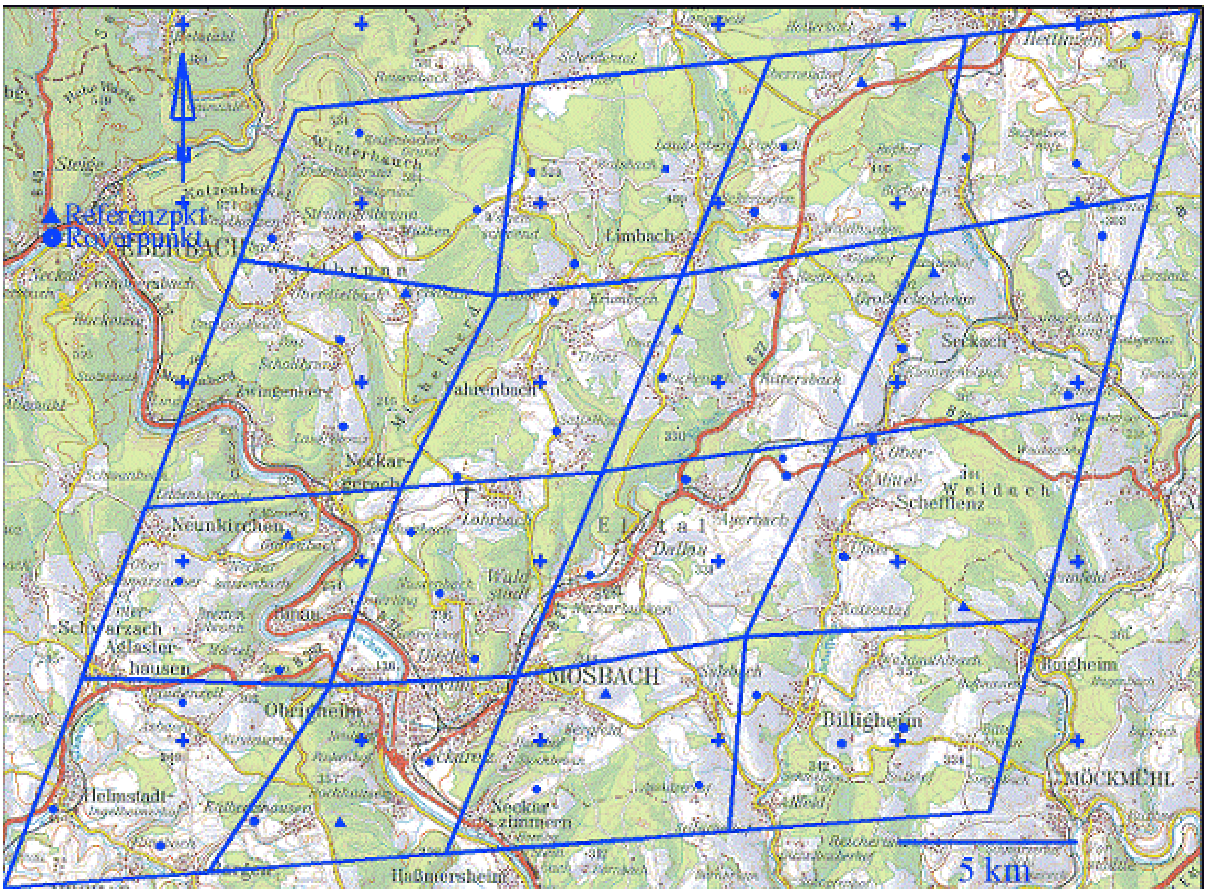

Fig. 7: Meshing and data design (H,h) of a DFHRS computation for the Mosbach region.

This requires however that the DGPS customer transmits (B,L,h) to the DGPS-service and gets back the corrected value H. The other way is of course to sell - like usual in the context with modern geoid-models [3] – the DFHRS data base directly to the DGPS user.

The first experiences with the DFHRS concept (12a,b,c) are much promising [12],[1]. As in most cases geoid information is available [3], the DFHRS evaluation may in general be set up together with a „geoid mapping“ (12b). For this complete case (12a,b,c) the best quality and control of the DFHRS will be achieved and at the same time the number of indentical points (h,H) for control and datum parameter estimation remains small even for large areas. The development of comfortable software for the production of DFHRS data bases is continued, and consequently also the implementation of DFHRS data bases in DGPS online software packages [13], [14].

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