DEFORMATION ANALYSIS WITH MODIFIED KALMAN-FILTERS

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Abstract

Kalman filtering can be considered a suitable tool for deformation analysis as it is a multiple-input, multiple-output digital filter that can optimally estimate, in real time, the states of a system based on its noisy outputs. The states are all the variables needed to completely describe the system behaviour of the deformation process as a function of time (such as position, velocity, ...). One can think of the multiple noisy outputs as a multidimensional signal plus noise with the system states being the desired unknown signals. The Kalman filter filters the noisy measurements to estimate the desired signals. The estimates are statistically optimal as they minimize the mean square estimation error.

The standard Kalman filter is a a low-pass filter. Therefore it can only be applied for the estimation of low frequency deformation signals. However, in many cases, the deformation process is described by a very complex spectrum of high and low frequency signals. With this publication will be shown, how modified Kalman filters can meet these requirements. The modified filters are based on the correlation functions of the processes.

Measurements, taken with a multi sensor hydrostatic levelling system in a nuclear power station, are used to clarify the concepts.

1. Constructions as dynamical systems

Constructions, as industrial or residential buildings, bridges, towers, influenced by internal or external forces, can be considered a linear lumped parameter system represented by a first order vector matrix differential equation

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{F}(t)\boldsymbol{y}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) + \boldsymbol{D}(t)\mathbf{w}(t) \tag{1}$$

where $\boldsymbol{y}(t)$ is the system state vector, $\boldsymbol{\omega}(t)$ is a random forcing function, $\boldsymbol{u}(t)$ is a deterministic (control) input, an $\boldsymbol{F}(t)$, $\boldsymbol{G}(t)$, $\boldsymbol{L}(t)$ are matrices arising in the formulation. Normally, this approach to system description is closer to physical reality than any of the frequency-oriented transformation techniques. It is particularly useful in providing statistical descriptions of system behaviour, as will be shown.

The state vector of the dynamic system shall be composed of any set of quantities sufficient to completely describe the unforced motion of that system. Given the state vector at a particular time

and a description of the system forcing and control functions from that point in time forward, the state of any other time can be predicted.
Sometimes, there is no deterministic input or this input is not well known. Then equ. (1) can be written as

$$\dot{y}(t) = \boldsymbol{F}(t)\boldsymbol{y}(t) + \boldsymbol{D}(t)\mathbf{w}(t) \tag{2}$$

2. Development of a discrete Kalman filter

Frequently deformation processes can be approximated by a logarithmic function superimposed by periodic random signals.

Periodic random functions can be described by the autocorrelation function (fig. 1)

$$\boldsymbol{C}^k_{yy\tau} = \sigma^2_w \boldsymbol{R}^k_{yy\tau} = \sigma^2_w e^{-\beta w \tau} \cos \omega_w \tau \tag{3}$$

with

σ : variance
ω : frequency
β : damping factor.

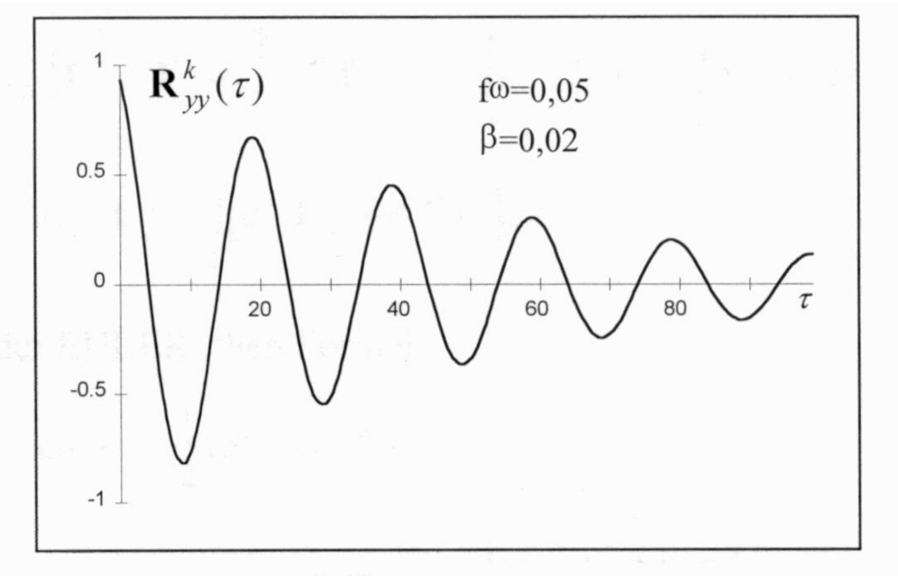

Fig. 1

With equ. (3) we get by applying Laplace Transformations the 2nd order linear differential equation [Ma 00]

$$\ddot{y}(t) + 2\beta\dot{y}(t) + \left(\beta^2 + \omega^2\right)\boldsymbol{y} = \mathbf{w}(t) \tag{4}$$

Now equ. (4) can be transformed into a set of n first-order linear differential equations, which can be expressed in matrix form as in equ. (2). With analytical or numerical methods finally we get the transition matrix and the covariance matrix of the system noise from the first order vector matrix differential equation (2) [Ge 74]. Usually the assumption has to be made that $\boldsymbol{F}$ is a matrix with constant coefficients and that $\boldsymbol{D}$ is random with mean zero over a certain time span.

However, these assumptions normally are not restrictive. $\boldsymbol{F} = \boldsymbol{const}.$ can always be achieved by breaking up the total time interval into pieces Δt for which this condition is satisfied. The second is a reasonable assumption in case all the major error sources have been modeled. Finally the prediction equation of the kalman filter and the covariance matrix of the system noise are given by [Ma 00]:

$$\begin{bmatrix} \boldsymbol{x}_{i,t+1}^k \\ \dot{\boldsymbol{x}}_{i,t+1}^k \end{bmatrix} = \begin{bmatrix} e^{-\beta\Delta t}\left(\cos\omega\Delta t + \frac{\beta}{\omega}\sin\omega\Delta t\right)\mathbf{I} & \frac{1}{\omega}e^{-\beta\Delta t}\sin\omega\Delta t\,\mathbf{I} \\ -\dfrac{\beta^2+\omega^2}{\omega}e^{-\beta\Delta t}\sin\omega\Delta t\,\mathbf{I} & e^{-\beta\Delta t}\left(\cos\omega\Delta t - \frac{\beta}{\omega}\sin\omega\Delta t\right)\mathbf{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{i,t}^k \\ \dot{\boldsymbol{x}}_{i,t}^k \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{\varepsilon,t}^k \\ \dot{\mathbf{w}}_{\varepsilon,t}^k \end{bmatrix} \tag{6a}$$

$$\boldsymbol{y}_{i,t+1}^k = \boldsymbol{T}_{i,\beta,\omega}^k \boldsymbol{y}_{i,t}^k + \mathbf{w}_{i,t}^k \tag{6b}$$

$$\overline{\Sigma}_{ww,t}^k = 4\left(\sigma_w^k\right)^2 \beta \int_0^{\Delta t} e^{-2\beta\tau} \begin{bmatrix} \begin{matrix} \sin^2\omega\tau \\ +\frac{\beta}{\omega}\sin\omega\tau\ \cos\omega\tau \end{matrix} & \begin{matrix} -2\beta\sin^2\omega\tau \\ +(\omega-\frac{\beta^2}{\omega})\sin\omega\tau\ \cos\omega\tau \end{matrix} \\ \begin{matrix} -\beta\sin^2\omega\tau \\ +\beta\cos^2\omega\tau \\ +(\omega-\frac{\beta^2}{\omega})\sin\omega\tau\ \cos\omega\tau \end{matrix} & \begin{matrix} 2\beta^2\sin^2\omega\tau \\ +(\omega^2-\beta^2)\cos^2\omega\tau \\ +(\frac{\beta^3}{\omega}-3\beta\omega)\sin\omega\tau\ \cos\omega\tau \end{matrix} \end{bmatrix} d\tau\ . \tag{7}$$

Sometimes the damping parameter β in equ. (3) can be chosen zero. Then the autocorrelation function (3) can be written as

$$\boldsymbol{C}_{yy\tau}^k = \cos\omega\tau$$

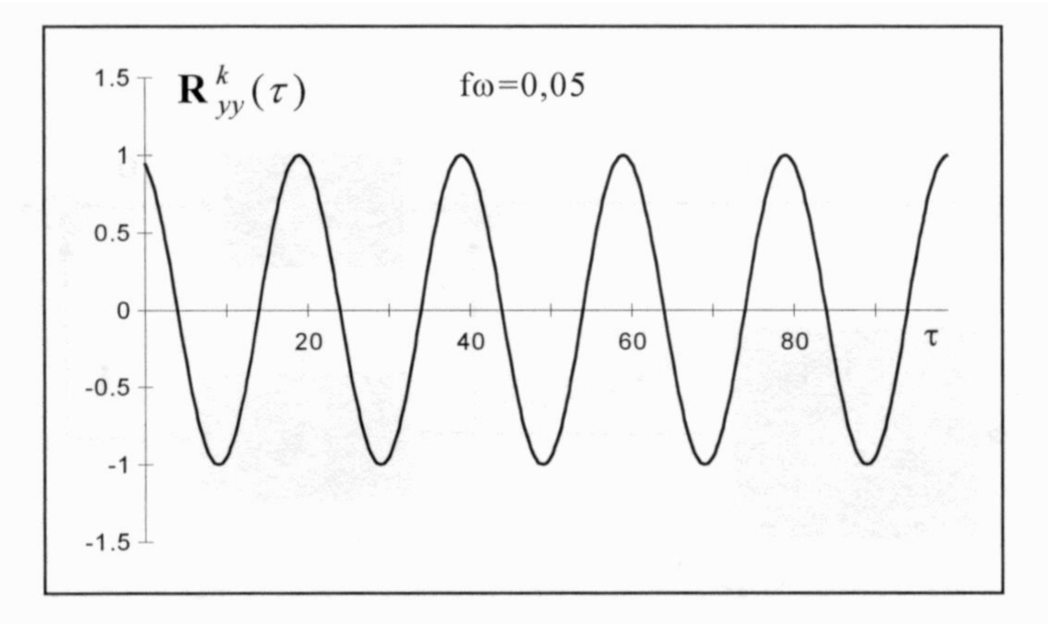

Fig. 2

Then the transition matrix and the covariance matrix are given by

$$\lim_{\beta\to 0} \boldsymbol{T}^k_{i,\beta,\omega,\Delta t} = \boldsymbol{T}^k_{i,\omega,\Delta t} = \begin{bmatrix} \cos\omega\Delta t\,\mathbf{I} & \frac{1}{\omega}\sin\omega\Delta t\,\mathbf{I} \\ -\omega\sin\omega\Delta t\,\mathbf{I} & \cos\omega\Delta t\,\mathbf{I} \end{bmatrix} \tag{9}$$

$$\overline{\Sigma}^k_{ww,t} = \left(\sigma^k_w\right)^2_i \begin{bmatrix} \frac{\Delta t}{2\omega} - \frac{\sin 2\omega\Delta t}{4\omega^3} & \frac{1-\cos 2\omega\Delta t}{4\omega^2}\mathbf{I} \\ \frac{1-\cos 2\omega\Delta t}{4\omega^2}\mathbf{I} & \frac{\Delta t}{2} + \frac{\sin 2\omega\Delta t}{4\omega}\mathbf{I} \end{bmatrix} \tag{10}$$

The long time trend of the deformations in general must also be considered a random signal which can be approximated by a logarithmic function. Normally these deformation can be described by a 3rd order system model, where the transitions matrix has the form

$$\boldsymbol{T}^k_{i,\Delta t} = \begin{bmatrix} \mathbf{I} & \Delta t\,\mathbf{I} & (\Delta t)^2\,\mathbf{I} \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix}. \tag{11}$$

As different empirical test have shown, then the variance s^2 of the covariance matrix of the system noise can be chosen zero [Ka 93].

The discrete state vector, describing the long time processes and periodical evenent, can then be written as follows

$$\begin{bmatrix} \boldsymbol{y}_{t+1} \\ \boldsymbol{y}^k_{1,t+1} \\ \vdots \\ \boldsymbol{y}^k_{m,t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \overline{\boldsymbol{T}}^k_{1,\beta_1,\omega} & \cdots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \overline{\boldsymbol{T}}^k_{m,\beta_m,\omega} \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{y}^k_{1,t} \\ \vdots \\ \boldsymbol{y}^k_{m,t} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_t \\ \mathbf{w}^k_{1,t} \\ \vdots \\ \mathbf{w}^k_{m,t} \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} y_{t+1} \\ \tilde{y}^k_{t+1} \end{bmatrix} = \begin{bmatrix} T_\alpha & 0 \\ 0 & T^k_{\beta,\omega} \end{bmatrix} \begin{bmatrix} y_t \\ \tilde{y}^k_t \end{bmatrix} + \begin{bmatrix} \mathrm{w}_t \\ \mathrm{w}^k_t \end{bmatrix} \tag{13}$$

$$\boldsymbol{y}^e_{t+1} = \boldsymbol{T}^e_{\alpha,\beta,\omega}\boldsymbol{y}^e_t + \mathbf{w}^e_t \tag{14}$$

3. Applications and results

Since 1988 the groundplate of a cooling tower was monitored because of larger settlements and the measurements were subject to scientific investigations. The deflections were monitored by a hydrostatic leveling system with 85 sensors. The sensors were equally devided along the deformation area, which stretched as a 40 m broad strip across the groundplate. The diameter of the ground-plate is 130 m.

To stop the deflections, systems of pipes were horizontally installed beyond the deformation area in a depth of 10 m. The pipes were used to pump an especial cement emulsion into the ground to press the groundplate back into its original form as far as possible.

Figure 3 gives an overview about the time series of the deflections, monitored between 1988 and 1993. The upper boundary line of the graph describes deflections of a point with smallest movements and the lower one a point with largest deflections. The time series show that depending on the deformation area the deflection rates vary between 1.5 to 4 cm per year.

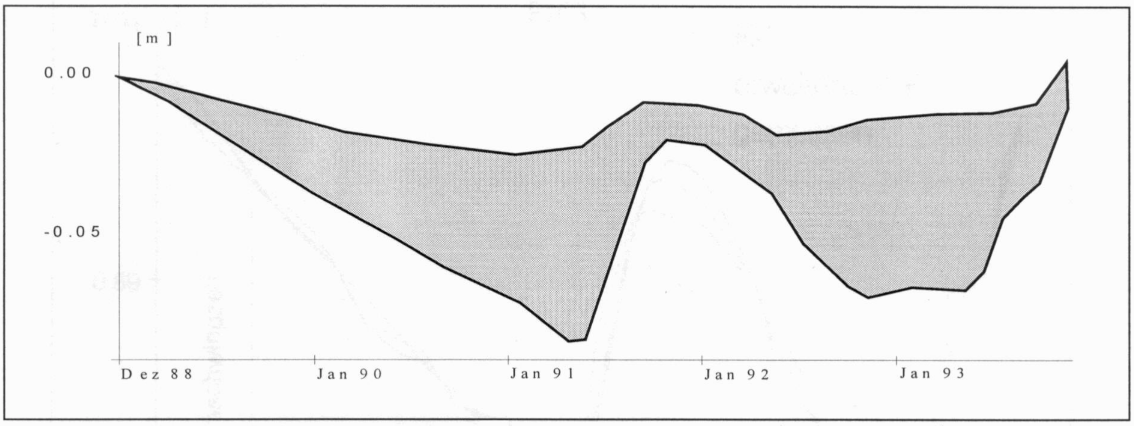

Fig. 3

In the beginning of 1991 the injection work was started and the groundplate could nearly be forced back into its original form. Then, after the injection process, in January 1992 there were again settlements till after 1993 other injections were performed. The long time deflections were superimposed by short time ones. The period of the short periodic signals was about one year and the amplitudes varied between 1 to 3 mm. There were maxima in winter time and minima in summer.

The main goal of the investigations was to separate the different signals so that the injections could be arranged more precisely and to detect the sources of the deflections.

In the following some results will be presented. Fig. 4 shows isolines of settlement rates and fig. 5 isolines of heave rates of the short periodic deflections in spring and fall 1990. The graphical representations show the high quality of the filters. Even isolines with very low deflection rates could be detected very precisely. Both graphs show that the source of the short periodic deflections can be located near the center of the groundplate.

Fig. 6 shows isolines of settlement rates in spring 1990. The black lines represent velocities after the short periodic signals were eliminated and the grey lines show isolines we got from the original signals. We can see that the isolines were shifted up to some metres, after the short periodic distortions were sliminated from the trend. This is especially the case for isolines with higher deflection rates. For the injection work this information is of great importance, as it should be based only on the isolines, caused by the long periodic trend. Another interesting fact is that here the sources of the deflections spread the deformation area.

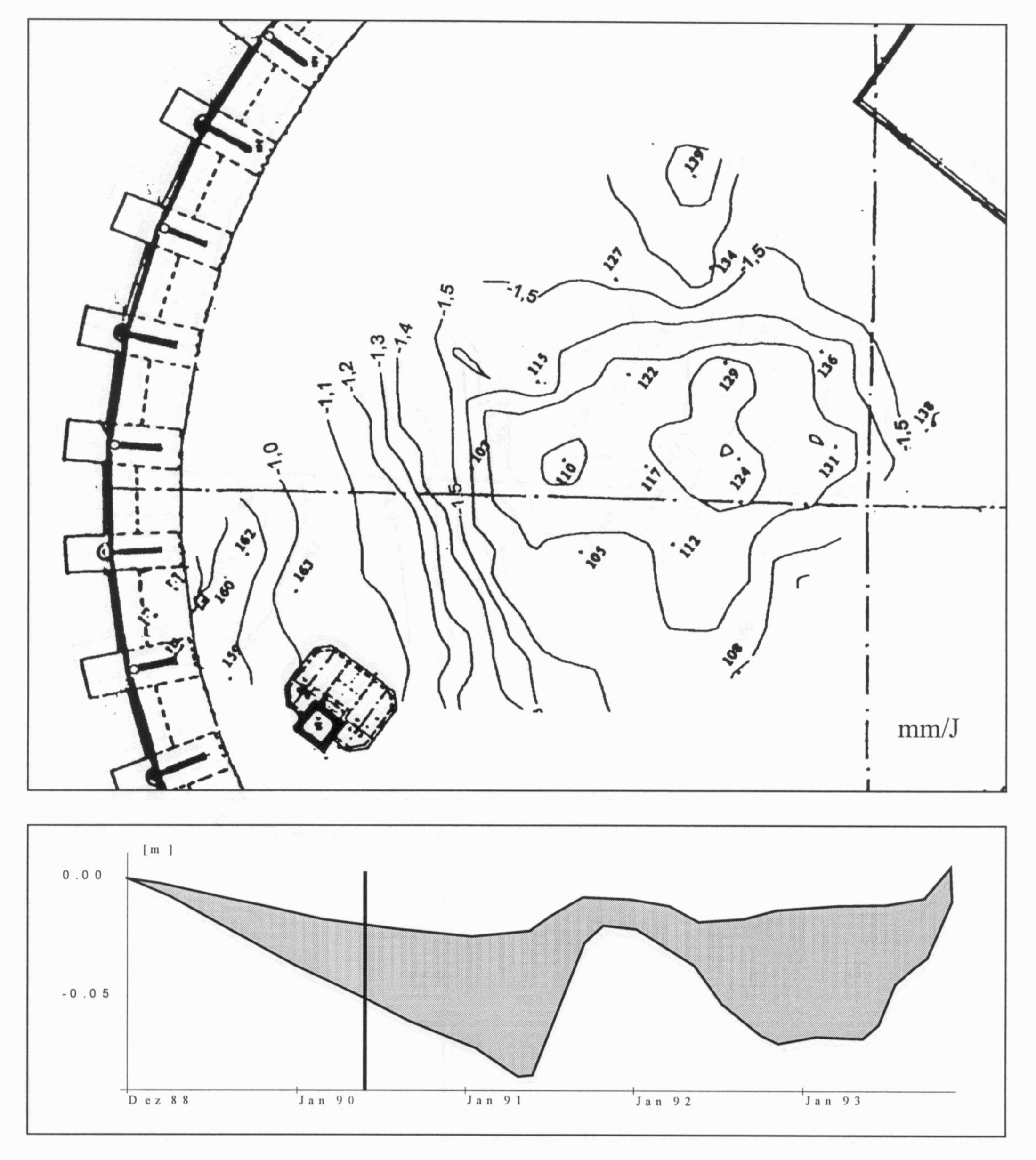

Fig. 4

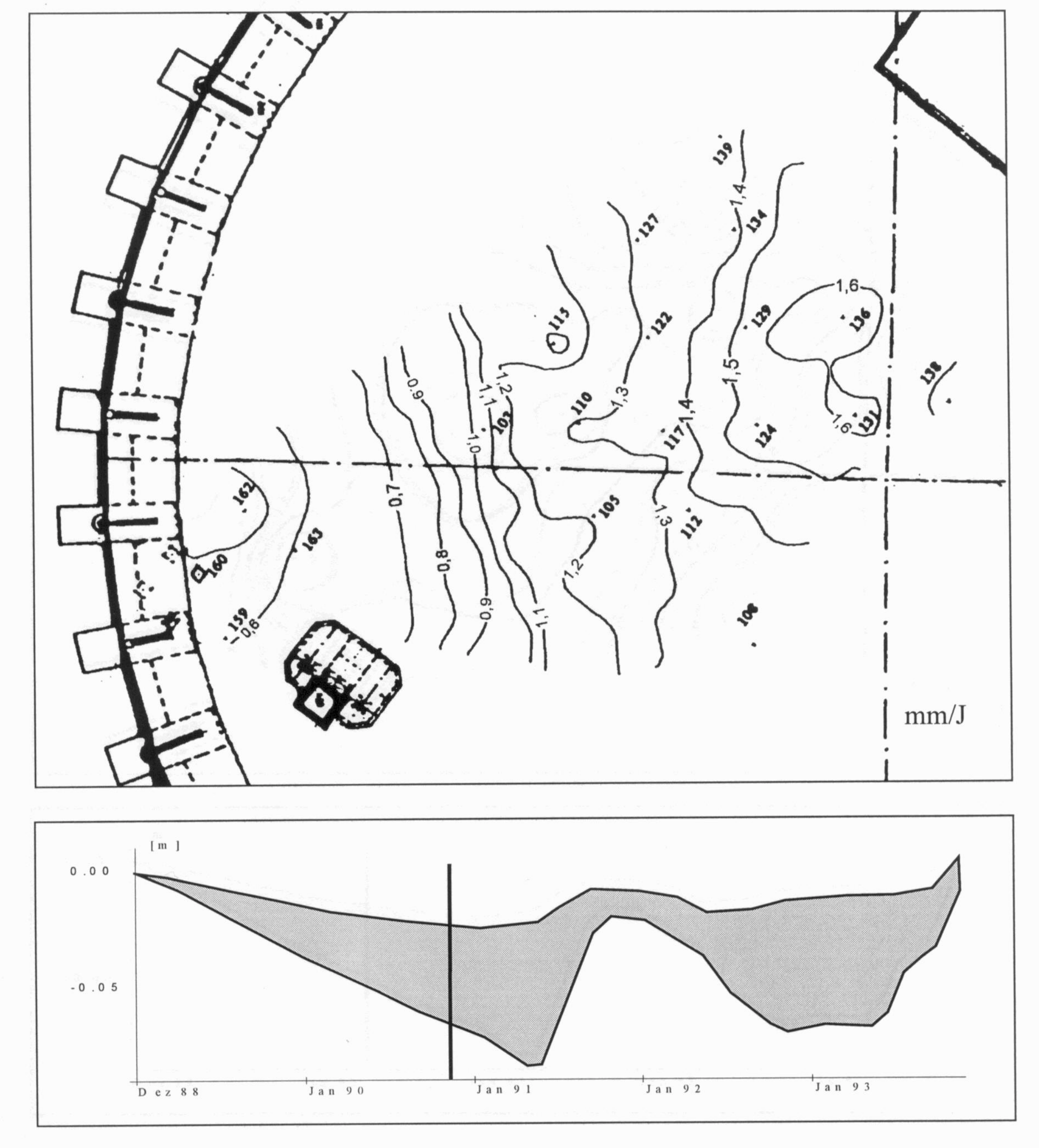

Fig. 5

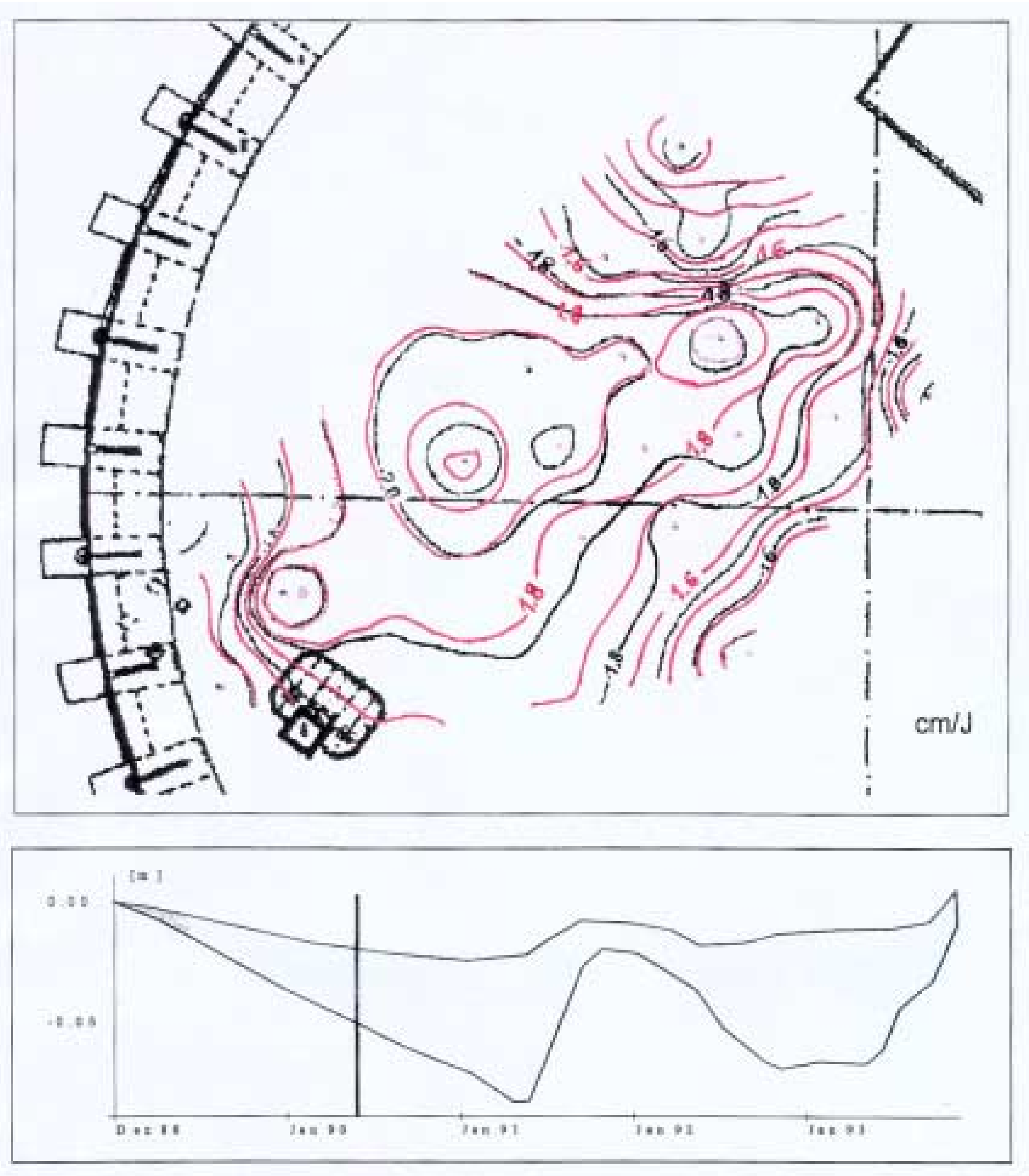

Fig. 6

4. Final remarks

It could be shown that with modified Kalman filters long periodic and short periodic signals of deflections can be separated very precisely. This is very helpful if irregular settlements of industrial buildings have to be replaced by injection work. The separation of the signals can also be used to find the sources of the deformations.

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