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KINEMATIC MODELING OF SUBSIDENCES

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Abstract

For the monitoring of subsidences observed in an area, a vertical control network is established. If several measuring epochs of the network's elements exist, a kinematic model is, subsequently applied for the description of the ground's behavior in the area of interest. The model is usually a function of the velocities and accelerations of the network's points or a polynomial describing a best fitting velocity surface. Its parameters (velocities and accelerations or coefficients of the polynomial) together with the points' heights valid for the reference epoch, are estimated through the simultaneous adjustment of all the measuring epochs for which the model is valid. Since the heights of the network's points as well as the model's parameters are valid for the reference epoch, for every other epoch they are estimated from the initial ones as prediction. Kalman filter technique is applied in order to update these parameters and control the validity of the model in use, every time a new measuring epoch takes place. In this paper a case study is presented, concerning the monitoring and modeling of subsidences disclosed in the quarter of Agia Triada in the city of Patras. Special emphasis is given in the application of Kalman filter technique for the updating of the model's parameters in order to investigate its capability (a) to confirm the validity of the model in use, and (b) to indicate the change of the kinematic behavior of the area, if any.

1. Introduction

After an earthquake that occurred in the northwest coast of Peloponnese in September 1989, a surface crack having a length of approximately 1.5km appeared in the quarter of Agia Triada in the city of Patras, and serious subsidences were observed. The crack coincides with the trace of one of the main existing tectonic faults of the region.
The Laboratory of General Geodesy (Rural and Surveying Engineering School of National Technical University of Athens) established a vertical control network in the area, in order to monitor the observed subsidences. The network consisted of 20 benchmarks, covering an area of 2.5 km^2, and 32 height differences between the network's points were determined by double leveling during each measuring campaign (Stathas et al., 1991). Ten measuring epochs took place in a total time interval of 6.5 years (February 1990 – October 1996). The automatic level NA2 – Wild was used for the observations of the first seven epochs, together with light staves having 1mm subdivisions, while, during the last three campaigns, the digital level NA2000 Wild was used together with bar coded staves. The measurements were carried out in a time span of four days, which, compared with the interval between two successive measurements of the network's elements, can be assumed as instant. Each measuring epoch was adjusted separately, the vertical displacements of the network's points were determined, and their statistical significance was tested. The estimated subsidences range up to 15cm for the total time interval.(Balodimos et al., 1991 - 1996)

2. Kinematic modeling of subsidences - Updating the model by Kalman filter technique

In order to represent the behavior of the subsidences, a kinematic model is usually applied (Holdahl 1978, Pelzer 1986). This model can be either a function of velocities and accelerations of the points' movements or a polynomial that defines a best-fitting velocity surface or, even, a combination of both. The observed height differences of the measuring epochs or the adjusted heights of the points can be used in a simultaneous adjustment for the determination of the model's parameters. The time intervals, for which the model is valid, are chosen after statistical tests and after studying the plotted diagrams of subsidences for all the network's points. The goodness of fit of the model and the significance of the estimated model's parameters is tested by applying special statistical tests.

It must be pointed out that all kinematic models determine the parameters of movement valid for the reference epoch t_0, which, usually coinsides with the first measuring epoch. For every other measuring epoch, heights and velocities of the points' movements can be calculated from the initial ones as **prediction**. It is, however, important to estimate the most **up to date** values of the parameters of movements in the area, especially when new observations of the network's elements take place. By applying Kalman filter technique, these updated parameters can be determined, and, consequently, the validity of the model in use is tested and changes of the area's kinematic behavior are determined as soon as possible (Pelzer 1986).

The kinematic state of the area (heights, velocities and accelerations of the points' movements) at epoch t_n ($t_n = t_0 + \Delta_t$) **is predicted** via the model, by the state vector:

$$\boldsymbol{y}(t_n) = \begin{bmatrix} \boldsymbol{H}^{t_n} \\ \boldsymbol{V}^{t_n} \\ \boldsymbol{a} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{I}\cdot\Delta t & \boldsymbol{I}\cdot\dfrac{\Delta t^2}{2} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{I}\cdot\Delta t \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{H}^{t_0} \\ \boldsymbol{V}^{t_0} \\ \boldsymbol{a} \end{bmatrix} = \boldsymbol{T}\cdot\boldsymbol{y}(t_0) \tag{2.1}$$

where:

$\boldsymbol{y}(t_0)$: the state vector at the reference epoch t_0 , and

$\boldsymbol{T}$: the transition matrix, which is function of time.

The variance – covariance matrix of the vector $\boldsymbol{y}(t_n)$ is given by:

$$\boldsymbol{V}_{y(t_n)} = \boldsymbol{T}\cdot\boldsymbol{V}_{y(t_0)}\cdot\boldsymbol{T}^{\mathrm{T}} = \hat{\sigma}^2_{0,m}\cdot\boldsymbol{T}\cdot\boldsymbol{N}_{y(t_0)}^{-1}\cdot\boldsymbol{T}^{\mathrm{T}} = \boldsymbol{T}\cdot\boldsymbol{Q}_{y(t_0)}\cdot\boldsymbol{T}^{\mathrm{T}} = \boldsymbol{Q}_{y(t_n)} \tag{2.2}$$

making the assumption that $\hat{\sigma}^2_{0,m} \cong 1$

If, at the same time t_n , a new measurement of the network's elements takes place, the observations can be used, applying the Kalman filter technique, for updating the state vector $\boldsymbol{y}(t_n)$, so that it represents the behavior of the area at this most recent epoch.

Using the recently made observations, the state vector $\boldsymbol{y}(t_n)$ can be written as:

$$\boldsymbol{l}^{t_n} + \boldsymbol{\upsilon}^{t_n} = \boldsymbol{A}\cdot\boldsymbol{H}^{t_n} = \begin{bmatrix} \boldsymbol{A}_H^{t_n} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{H}^{t_n} \\ \boldsymbol{V}^{t_n} \\ \boldsymbol{a} \end{bmatrix} = \boldsymbol{A}_{y(t_n)}\cdot\boldsymbol{y}(t_n) \tag{2.3}$$

Combining equations (2.1) and (2.3) the following observation equations are formed:

$$\begin{bmatrix} \boldsymbol{y}(t_n) \\ \boldsymbol{l}^{t_n} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\upsilon}_{y(t_n)} \\ \boldsymbol{\upsilon}_{l^{t_n}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{A}_H^{t_n} \end{bmatrix} \cdot \hat{\boldsymbol{y}}(t_n) \tag{2.4}$$

Then, Kalman filter technique is applied as follows (Pelzer 1986):

a. The vector $\boldsymbol{w}$ of differences between the carried out observations and their prediction through the kinematic model is calculated:

$$\boldsymbol{w} = \boldsymbol{l}^{t_n} - \begin{bmatrix} \boldsymbol{A}_H^{t_n} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{H}^{t_n} \\ \boldsymbol{V}^{t_n} \\ \boldsymbol{a} \end{bmatrix} = \ddot{A}\boldsymbol{H}_{ij}^{t_n} - \begin{bmatrix} \boldsymbol{A}_H^{t_n} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{H}^{t_n} \\ \boldsymbol{V}^{t_n} \\ \boldsymbol{a} \end{bmatrix} \tag{2.5}$$

b. The corrections $\boldsymbol{\Delta y}(t_n)$ that must be applied to the predicted state vector $\boldsymbol{y}(t_n)$ $(= \boldsymbol{T} \cdot \boldsymbol{y}(t_0))$ are computed.

$$\boldsymbol{\Delta y}(t_n) = \boldsymbol{G} \cdot \boldsymbol{w} \tag{2.6}$$

$$\boldsymbol{G} = \boldsymbol{Q}_{y(t_n)} \cdot \boldsymbol{A}_{y(t_n)} \cdot \left[\boldsymbol{P}_{l^{tn}}^{-1} + \boldsymbol{A}_{y(t_n)} \cdot \boldsymbol{Q}_{y(t_n)} \cdot \boldsymbol{A}_{y(t_n)}^{\mathrm{T}} \right]^{-1} \tag{2.7}$$

where:

$\boldsymbol{G}$: the gain matrix of Kalman filter technique,

$\boldsymbol{G}$ $\boldsymbol{Q}_{y(t_n)}$: the matrix of weight coefficients of the state vector $\boldsymbol{y}(t_n)$ as determined through equation (3.2), and

$\boldsymbol{P}_{l^{tv}}$: the observations' weight matrix.

c. Finally, the updated state vector $\hat{\boldsymbol{y}}(t_n)$ is computed:

$$\hat{\boldsymbol{y}}(t_n) = \boldsymbol{y}(t_n) + \boldsymbol{\Delta y}(t_n) \tag{2.8}$$

Thus, the most recent information about the behavior of the area, being present in the observation vector, is incorporated in the state vector.

The covariance matrix of the correction vector $\boldsymbol{\Delta y}(t_n)$ as well as that of the updated vector $\hat{\boldsymbol{y}}(t_n)$ are given by the equations:

$$\boldsymbol{V}_{\Delta y(t_n)} = \boldsymbol{Q}_{\Delta y(t_n)} = \boldsymbol{G} \cdot \left[\boldsymbol{P}_{l^{tn}}^{-1} + \boldsymbol{A}_{y(t_n)} \cdot \boldsymbol{Q}_{y(t_n)} \cdot \boldsymbol{A}_{y(t_n)}^{\mathrm{T}} \right] \boldsymbol{G}^{\mathrm{T}} \tag{2.9}$$

$$\boldsymbol{V}_{\hat{y}(t_n)} = \boldsymbol{Q}_{\hat{y}(t_n)} = \left[\boldsymbol{Q}_{y(t_n)} - \boldsymbol{Q}_{\Delta y(t_n)} \right] \tag{2.10}$$

d. The significance of the vector's $\boldsymbol{\Delta y}(t_n)$ elements is tested for significance level 95%, by applying the following statistical test:

$$\left(\left| \frac{\Delta y_i(t_n)}{\sigma_{\Delta y_i(t_n)}} \right| \le z_{0.95} \right) \tag{2.11}$$

In the case that the elements prove to be statistically insignificant, it is assumed that the model continues to represent the behavior of the area. The parameters of movement are, therefore, given by the vector $\boldsymbol{y}(t_n)$. Otherwise, it is concluded that the behavior of the area has changed, and is described by the updated vector $\hat{\boldsymbol{y}}(t_n)$.

3. Updating the kinematic parameters of the study area by Kalman filter technique

In this case, Kalman filter technique was used in order to investigate its capability not only to confirm the validity of the model in use, but, also, to indicate the changes in the kinematic behavior of the area under consideration, if any.

Thus, two distinctly different cases were considered:

a. In the first one a kinematic model (Balodimos et al. 1993) was applied for the time interval March 1991 – October 1995. The measured height differences of the seven measuring campaigns that took place during this time interval, were adjusted simultaneously and heights, velocities and accelerations of the network's points were determined for the reference epoch t_0 =March 1991. The observation equations of the simultaneous adjustment were of the form:

$$\Delta H_{ij}^{t_n} + \upsilon_{ij}^{t_n} = H_j^{t_0} - H_i^{t_n} + \left(V_j - V_i\right)\cdot\left(t_n - t_0\right) + \left(a_j - a_i\right)\cdot\frac{\left(t_n - t_0\right)^2}{2} \tag{3.1}$$

$\Delta H_{ij}^{t_n}$: the observed height difference between points *i, j* of the network at measuring epoch t_n,

$\upsilon_{ij}^{t_n}$: the corresponding residual,

$H_i^{t_0}, H_j^{t_0}$: the heights of the above mentioned points at the reference epoch t_0,

V_i, V_j : the (initial) velocities of the points' vertical displacements,

a_i, a_j : the corresponding accelerations of the points' movements, and

$t_n - t_0$: the time interval between measuring epoch t_n and the reference epoch t_0.

Kalman filter technique was applied, using the results of the measuring campaign that took place one year later (October 1996) in order to update the parameters of the model. The vector of corrections of the predicted parameters was calculated, the statistical significance of its elements was tested, and finally the updated parameters of the model were computed. The results are depicted in Table 1. The predicted and updated velocity surface is plotted in Fig. 1 and 2 respectively.

OCTOBER 1996						
	HEIGHTS (m)		**VELOCITIES** (mm/year)		**ACCELERATIONS** (mm/year^2)	
B.M.	PREDICTION	UPDATING	PREDICTION	UPDATING	PREDICTION	UPDATING
14	23.679	23.681	-2	-2	0	0
13	20.613	20.615	1	2	2	2
103	21.039	21.041	1	2	2	2
16	21.542	21.542	0	0	0	0
17	18.234	18.236	3	3	2	2
18	16.809	16.810	5	5	4	3
26	16.328	16.330	0	0	0	0
36	16.996	17.001	-1	0	0	0
19	13.213	13.212	7	5	4	4
25	14.019	14.015	10	6	7	6
106	8.381	8.383	3	3	2	2
20	10.584	10.586	2	2	1	1
24	9.730	9.723	14	9	7	6
3	8.024	8.025	3	2	1	1
21	5.795	5.791	11	8	7	6
2	2.369	2.370	4	3	1	1
23	2.765	2.767	4	4	2	2
22	2.370	2.372	6	5	4	4

Table 1. October 1996 - Predicted and updated parameters of the kinematic model.

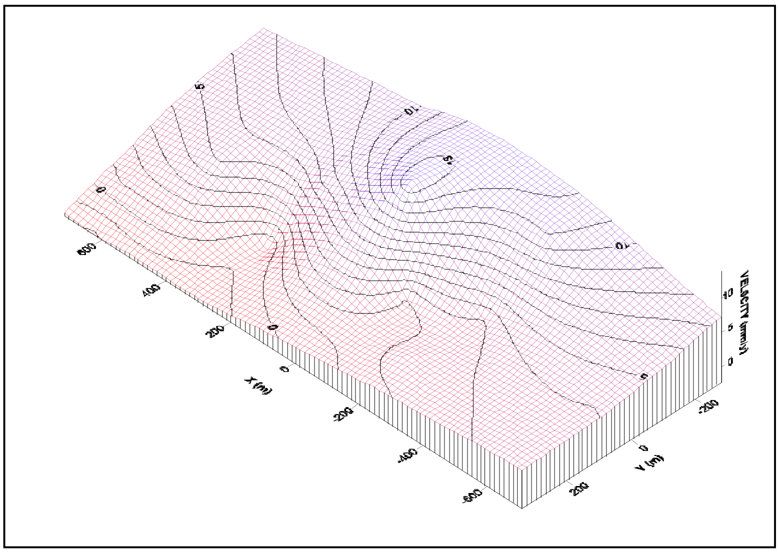

Fig.1 October 1996 – Predicted velocity surface

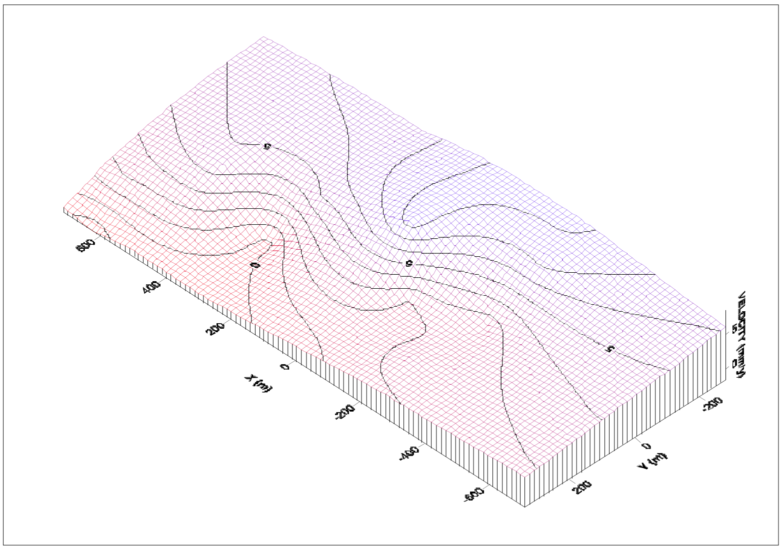

Fig. 2. October 1996 – Updated velocity surface

From the contents of Table 1 as well as from the plotted 3-D velocity surfaces it can be noticed that the differences between prediction and updating are practically insignificant. It is therefore concluded that the kinematic behavior of the study area is still represented by the model and the parameters of the model for October 1996 are given through the state vector of equation (2.1).

b. The same kinematic model was applied for the time interval March 1991 – December 1992, by adjusting simultaneously the measured height differences of the five measuring campaigns that took place in the above mentioned time interval. Kalman filter technique was applied, using the observations of July 1993 campaign, in order to update the model's parameters. The results are depicted in Table 2. The corresponding predicted and updated 3-D velocity surfaces are plotted in Fig. 3 and 4 respectively.

JUNE 1993						
	HEIGHTS (m)		**VELOCITIES** (mm/year)		**ACCELERATIONS** (mm/year^2)	
B.M.	PREDICTION	UPDATING	PREDICTION	UPDATING	PREDICTION	UPDATING
14	23.685	23.686	-5	-4	-2	-2
13	20.614	20.617	-15	-12	-7	-5
103	21.040	21.044	-18	-15	-8	-7
16	21.542	21.542	0	0	0	0
4	22.589	22.589	0	0	0	0
17	18.228	18.237	-25	-13	-13	-6
18	16.796	16.810	-37	-19	-19	-8
26	16.327	16.327	0	0	0	0
36	16.998	16.998	-2	-2	0	0
37	14.198	14.210	-36	-18	-18	-7
19	13.194	13.212	-46	-23	-24	-10
25	14.003	14.020	-53	-36	-23	-16
106	8.378	8.379	-5	-4	0	0
20	10.579	10.583	-11	-7	-5	-3
24	9.701	9.721	-50	-31	-22	-14
3	8.017	8.020	-9	-5	-5	-2
21	5.775	5.792	-49	-32	-22	-14
2	2.358	2.364	-11	-4	-5	-2
23	2.756	2.763	-19	-12	-8	-6
22	2.353	2.371	-43	-21	-22	-10

Table 2. June 1993 - Predicted and updated parameters of the kinematic model.

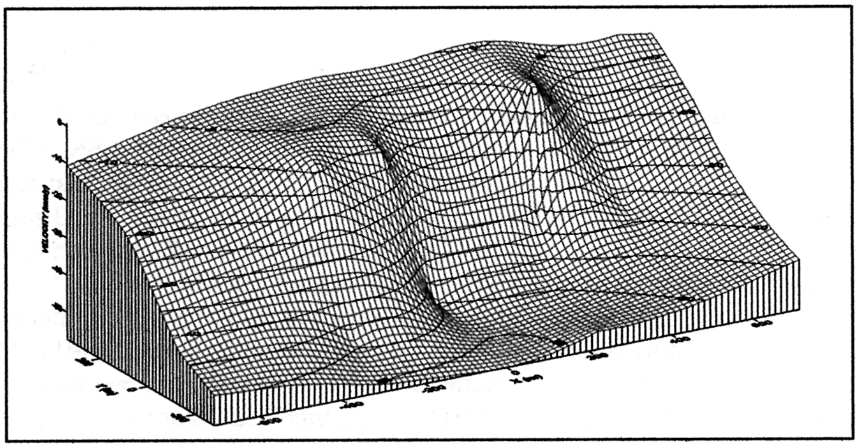

Fig.3. June 1993 – Predicted velocity surface

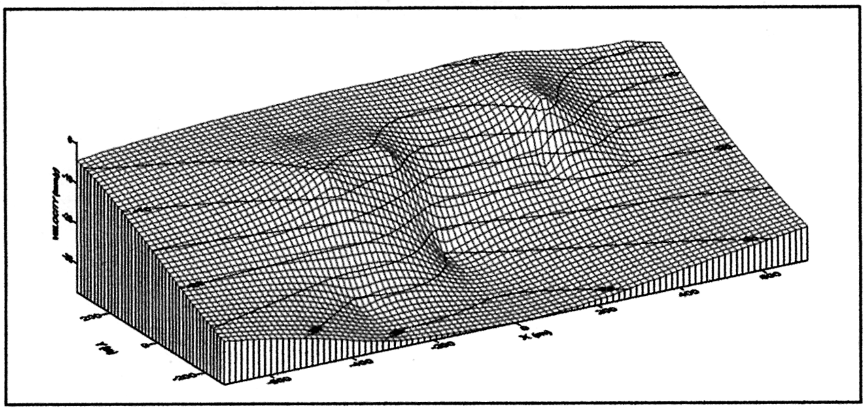

Fig. 4. June 1993 - Updated velocity surface

From the results depicted in Table 2, as well as from the shapes of the predicted velocity and surface versus the updated one, it can be seen that significant differences exist between prediction and updating. Thus it can be concluded that the kinematic behavior of the area under investigation has changed in the time interval December 1992 – June 1993, and the parameters of the model for June 1993 are the, via Kalman filter technique, updated ones (Columns 3, 5 & 7 of Table 2).

4. Conclusions

From the results obtained through the application of Kalman filter technique in the above case study, the following conclusions are withdrawn:

- Kalman filter technique is an adequate and reliable tool for the determination of the most up to date kinematic parameters of the area under consideration.
- As it has been proved by the results it is possible, not only to confirm the validity of the model in use, but also, to detect any change of the kinematic behavior of the area, as soon as possible.
- In cases of underground works such as tunneling, especially in urban areas, where it is of utmost interest not only to detect, as soon as possible, the expected ground subsidences, but also to monitor and even predict their behavior in order to prevent damages to structures and utilities at ground surface, repeatedly measured vertical control networks are established. By applying a kinematic model, using the results of the first four of five measuring campaigns the parameters of the ground movements can be estimated. The application of Kalman filter technique, every time a new measuring epoch takes place, can then provide the most up to date kinematic parameters of the area, and thus give an early warning signal if they exceed the expected values.

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