Evaluation of GPS Measurements of Railway Track Geometric Position

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Key words

SUMMARY

Application of GPS method in the measurements of railway track geometrical position is interesting from the point of view of reached accuracy and the methodology of adjustment the observed data with respect of its evaluation in local coordinate system, so called topocentric horizontal coordinate system.

However, it is important to compare the precision of terrestrial surveying and GPS observation of the railway track used in a local coordinate system.

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1. INTRODUCTION

Global Positioning System as a surveying technology has found wide possibilities of its applying in Engineering Geodesy, since. Only, the restraining factor of GPS use in engineering surveying can be non-performance of a satellite availability and visibility and non-fulfillment of the defined accuracy varied on a character of a work.

One of the possibilities of GPS use in building engineering, especially in transport building was application the static and kinematic GPS method in control measurement of a part of railway line to determine the track geometrical position. These control observations were realized in experimental locality, which consists of regional railway, and were done under regular traffic.

The satellite visibility was not provided on the whole length of the railway line and therefore the GPS technique had to combined with terrestrial measurements.

The geometrical track position was determined in relation to the local reference system with seven points of a permanent deep stabilization in a form of “small” triangulation network (Fig. 1). So, the precision and the quality of the track position determination depend direct on the accuracy and quality of this local network. Therefore, the accuracy analysis of control measurements comprises of two steps:

- Evaluation of the accuracy and stability of the network measured with the both of methods: GPS-static and terrestrial.
- Evaluation of surveying accuracy of the railway track geometrical position, which was measured with GPS-kinematic and intersection method.

Hence, the control measurements of the track geometrical position have predefined accuracy, which is included in the particular technical standards (1), (2), (3), paper is aimed to describe the adjustment procedure both of observations, especially from the point of view of accuracy analysis that can be the prevailing argument in potential decision making upon using suitable adjustment method.

First part of the paper deals with practical measurements realized in local network and upon the railway line. GPS and terrestrial techniques were combined because of fulfillment GPS observations with insufficient visibility.

In the second part, the adjustment process is explained. Combine adjustment procedure were chosen to secure the unify precision of determination the track position in the whole length. This type of adjustment procedures assumes to determine weight components of measured data with e.g. MINQUE method of variance component estimation.

The mathematical part of paper is enriched with tables consisted of experimental data and adjustment outputs to illustrate the theoretical declarations.

Adjustment process was realized in 2D coordinate system because of precision incomparability of positional and heights parameters of GPS measurement. The heights adjustment is the subject of another study.

2. GPS AND TERRESTRIAL SURVEY TECHNIQUES

Triangulation network was measured by static GPS survey technique and triangulation method.

Three single frequency GPS receivers were used to measure the GPS baseline distances. Maximum length of the baseline in the triangulation network was about 102 m, while the coordinate differences of each vector were calculated from the basis F4 (see figure 1). The observation time was 50 minutes in each point, the visibility was from 8 to 9 satellite and PDOP varied from 1 to 3. The precision of the antenna stabilization not exceeds one millimeter in each point because of their permanent deep stabilization.

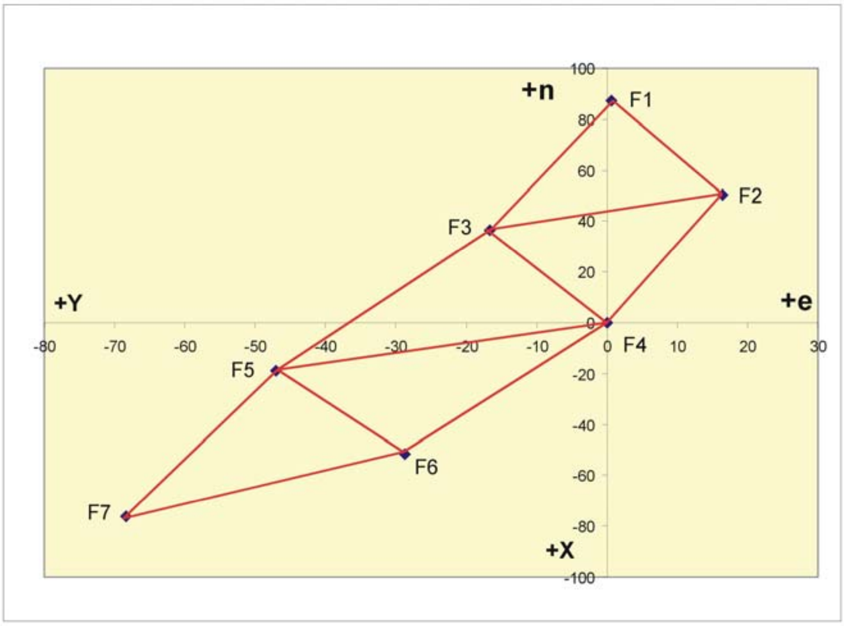

Figure 1: Triangulation network

The GPS observations were evaluated via post processing software, which determined the coordinate differences of the particular reference points, defined in WGS-84 geocentric ellipsoid model and the variance-covariance matrix, which represents a priori surveying precision.

In the experimental railway line, the track geometrical position was measured with Stop&go GPS technique simultaneously with classic intersection method. Observation time of the particular points laying on the track was 30 seconds and the totally observation time of the whole length of railway line takes 40 minutes. Result of GPS track observation consists of geocentric coordinate differences of the particular track points like the standard deviations as the precision parameters. The configuration of the railway line and triangulation network is shown in figure 2.

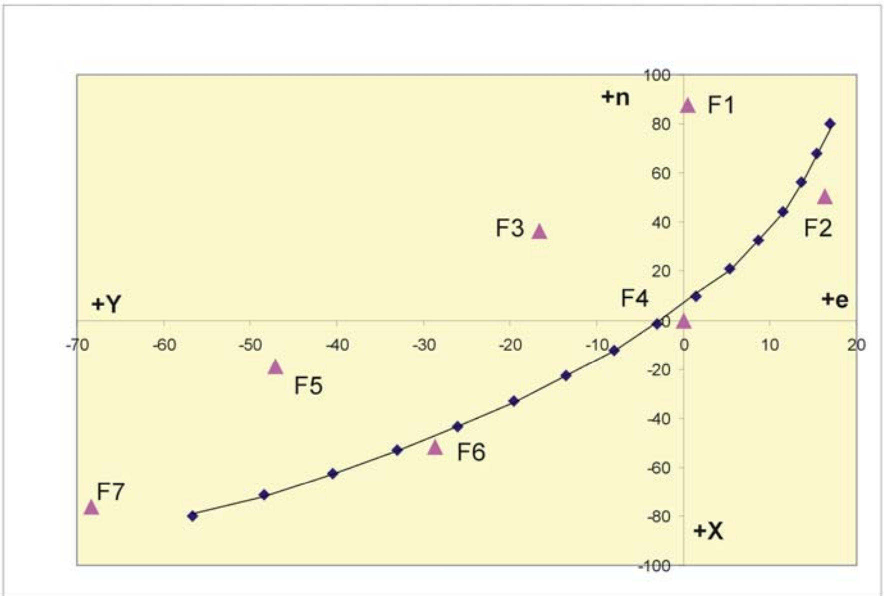

Figure 2: Railway track

Precision of the terrestrial observations represents the standard deviation of angles measurement, which is depended on the size (U) and number (t) of triangles misclosures and is calculated according to the Ferrer equation (4):

$$m_F^2 = \sqrt{\frac{\sum U^2}{3t}}\,.$$

3. ADJUSTMENT PROCESS

Adjustment process comes after GPS post processing and aims to determine the precise position of the triangulation points and consequently the position of the railway line.

Many technologies and methods are used to evaluate mixed independent observations related to the reference network (5), (6), (7). To unify of the precision of track position determination, we have preferred combined adjustment both of heterogeneous data represented by Cartesian coordinates of triangulation points find out by GPS method and angels and distance measured by classic terrestrial method.

Adjustment procedure started with transformation of Cartesian geocentric coordinate differences ($\Delta X, \Delta Y, \Delta Z$) into coordinates defined in the local topocentric system (n, e, v) described e.g. in the (8), (9) using relation:

$$\begin{pmatrix} n \\ e \\ v \end{pmatrix} = \mathbf{R}(B, L) \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix},$$

where the rotation matrix R is expressed as:

$$R = \begin{pmatrix} -\sin B \cos L & -\sin B \sin L & \cos B \\ -\sin L & \cos L & 0 \\ \cos B \cos L & \cos B \sin L & \sin B \end{pmatrix}.$$

Similarly, the variance-covariance matrix of the transformed parameters is determined as follows:

$$\mathbf{C}_{nev} = \mathbf{R} \,.\, \mathbf{C}_{X,Y,Z} \mathbf{R}^{\mathbf{T}}$$

So, the horizontal topocentric system creates free network with two known reference datum parameters: the GPS basis $\text{F4}\,(0{,}0)$ as the coordinate center and the azimuth of baseline σ_{46}.
For adjustment the local triangulation network we have applied Least Squares Adjustment Method with using the stochastic model, which assumes that errors in observations, and the residuals are normally distributed.

If the mathematical correlation is indicated (physical correlation, which arises from the nature and observation technology is not the theme of this study) after data processing, full variance-covariance matrix is assumed to enter into adjustment procedure, which is anticipated to be determined in GPS processing software. The converted variance-covariance matrix of baseline observation has the following structure:

$$\mathbf{C} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 \end{pmatrix} = \begin{pmatrix} 0{,}003 & 0 & 0 \\ 0 & 0{,}003 & 0 \\ 0 & 0 & 0{,}003 \end{pmatrix}.$$

The triangles configuration of the local network predicted to use combine case of conditional adjustment written in general form:

$$f(l, x) = 0 .$$

The condition equations are created by triangles conditions and by comparison both of types of coordinate differences of triangulation points, GPS and terrestrial, where the terrestrial coordinates represent the unknown parameters dx_i. The common condition equations after linearization are in form:

$$\mathbf{A}^{\mathbf{T}}\mathbf{Q}\,\mathbf{A}\mathbf{k} + \mathbf{B}\,\mathbf{dx} + \mathbf{u} = 0 ,$$

$$\mathbf{B}^{\mathbf{T}}\mathbf{k} = 0 ,$$

and the solution for the unknown parameters is:

$$\mathbf{dx} = \left(\mathbf{B}^{\mathbf{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{Q}\,\mathbf{A}\right)^{-1}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathbf{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{Q}\,\mathbf{A}\right)^{-1}(-\mathbf{u}),$$

where **A** and **B** are design matrices containing the partial derivatives of the observations and unknown components, and **u** is the vector of misclosures, which is expressed in a general form:

$$\mathbf{u} = f(l', x_0).$$

Stochastic model of adjustment assumes to know covariance matrix of the observation as the inverse of weight matrix $\mathbf{Q} = \mathbf{P}^{-1}$. Hence, the entrance values are independent **Q** is diagonal matrix.

If the observations are heterogeneous, the weight parameters are defined in respect of the origin of this incoming data by the variance component estimation (VCE) described e.g. in (10), (11), (12), (13).

4. VARIANCE COMPONENT ESTIMATION

One of the commonly used methods for estimation of variance-covariance components of GPS observations is MINQUE method (Minimum Norm Quadratic Unbiased Estimation).

MINQUE as known technique for estimating precision of different observations have been entered into the adjustment process by analyzing the estimates residuals **v** of the observations, which are arranged to linearized Gauss-Markoff-Model:

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v}\,.$$

Generally, VCE helps to estimates parameter θ of covariance matrix by iterative procedure, which starts with following equation:

$$\mathbf{C} = \mathbf{P}^{-1} = \theta^{j}\ \mathbf{T}\,.$$

Providing, the components of weight matrix are expressed as

$$\mathbf{P} = \{\sigma_0 / \sigma_i\},$$

we can put firstly $\theta^0 = 1$ and the matrix $\mathbf{T} = \left\{\dfrac{\theta^{j}}{\sigma_i}\right\}$.

One of the adjustment products is vector of the residuals in the form:

$$\mathbf{v} = -\left(\mathbf{A}^{\mathbf{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathbf{T}}\mathbf{P}\,\mathbf{l}\,,$$

with an appropriate cofactor matrix of adjusted residuals:

$$\mathbf{Q}_{\mathbf{v}} = \mathbf{P}^{-1} - \mathbf{A}(\mathbf{A}^{\mathbf{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathbf{T}}\,.$$

Then the vector **q** can be calculated by multiplying following matrices:

$$\mathbf{q} = \mathbf{v}^{\mathbf{T}}\mathbf{R}\,\mathbf{T}\,\mathbf{R}\,\mathbf{v}\,,$$

where R is partitioned matrix calculated according to the equation:

$$\mathbf{R} = \mathbf{P}\,\mathbf{Q}_{\mathbf{v}}\mathbf{P}\,.$$

Using the previous estimate θ^{j} as the a priori value, the new estimate is:

$$\theta^{j+1} = \mathbf{S}^{-1}\mathbf{q}$$

and matrix $\mathbf{S}$ is a trace function expressed as:

$$\mathbf{S} = \{S_{ij}\}$$

$$S_{ij} = tr(\mathbf{R}\mathbf{T}_{\mathbf{i}}\mathbf{R}\mathbf{T}_{\mathbf{j}})$$

If θ converges, the limiting value of θ satisfies the following equation:

$$\mathbf{S}\theta = \mathbf{q}\,.$$

5. ACCURACY ANALYSIS

A prior variance σ_{0A} determined by MINQUE iteration procedure is displayed in table 1, which characterized the precision both of GPS coordinates Y_G, X_G and terrestrial Y_T, X_T entered into combined adjustment. According to the value σ_{0A} and the cofactor of terrestrial coordinates q_Y, q_X the variance components of measured angels and distance could be backwards calculated by applying the Law of Weight Propagation:

$$q_F = \mathbf{A}^{\mathrm{T}} \mathbf{Q} \mathbf{A}.$$

Table 1

Triangulation points	GPS measurements				Terrestrial measurements			
	A prior accuracy: $m_{0A} = 0{,}009\,\mathrm{m}$							
	Standard deviations		$Q = P^{-1} = \{q_{ii}\}$		Standard deviations		$Q = P^{-1} = \{q_{ii}\}$	
	m_Y	m_X	q_Y	q_X	m_Y	m_X	q_Y	q_X
F1	0,002	0,002	0,1	0,1	0,004	0,008	0,2	0,9
F2	0,002	0,002	0,1	0,1	0,003	0,007	0,1	0,5
F3	0,002	0,002	0,1	0,1	0,005	0,011	0,3	1,5
F4	0,002	0,002	0,1	0,1	0,002	0,002	0,1	0,1
F5	0,004	0,004	0,2	0,2	0,007	0,013	0,5	2,1
F6	0,002	0,002	0,1	0,1	0,009	0,015	0,9	2,8
F7	0,002	0,002	0,1	0,1	0,007	0,015	0,7	2,7

A posterior variance factor characterizes the accuracy of adjustment procedure and should be comparable with a prior value m_{0A}^2:

$$\sigma_{0AP} = \frac{\mathbf{v}^{\mathrm{T}} \mathbf{P}\, \mathbf{v}}{n - t}.$$

A posterior accuracy and the differences between adjusted GPS and terrestrial coordinates are shown in table 2.

Table 2

Triangulation points	A posterior accuracy: $m_{0AP} = 0{,}012\,\mathrm{m}$	
	v_Y in mm	v_X in mm
F1	-4.32	0.78
F2	2.90	1.25
F3	3.32	-4.87
F5	-0.48	1.61
F6	1.16	0.21
F7	-2.58	1.00

Accuracy of the terrestrial coordinates as the unknown parameters Y_{Ti}, X_{Ti} is represented by variance-covariance matrix of unknown parameters $\mathbf{Q}_{\overline{\mathbf{xx}}}$ expressed by equation expressed in (14).

$$\mathbf{Q_{xx}} = \left(\mathbf{B}^{\mathbf{T}}\left(\mathbf{A}^{\mathbf{T}}\mathbf{QA}\right)^{-1}\mathbf{B}\right)^{-1}.$$

In consideration of above describe accuracy analysis of local network the precision of the railway line lateral position is calculated in consequence.

The variance-covariance matrix represents the accuracy of determination of the particular points laying on the track and observed by kinematic GPS method. It was found out by the conversion of $\mathbf{C}_{XYZ}$ matrix, from the WGS-84 into topocentric horizontal system, which was evaluated in post processing software.

The precision of track position determination by intersection method is defined by standard deviations evaluated by applying the Law of Error Propagation. The comparison both of precisions concerned to 18 particular track points is expressed in the following table.

Table 3

GPS measurements				Terrestrial measurements			
m_Y		m_X		m_Y		m_X	
0,002	0,003	0,004	0,004	0,035	0,053	0,048	0,038
0,002	0,004	0,003	0,007	0,035	0,053	0,048	0,038
0,003	0,003	0,004	0,003	0,035	0,063	0,048	0,046
0,002	0,003	0,004	0,005	0,035	0,063	0,048	0,045
0,002	0,003	0,004	0,003	0,053	0,063	0,038	0,045
0,002	0,002	0,003	0,002	0,053	0,063	0,038	0,045
0,002	0,002	0,003	0,003	0,053	0,063	0,038	0,045
0,002	0,004	0,004	0,004	0,053	0,063	0,038	0,045

To compare the predefined accuracy contained in technical standard (1) table 4 is added, which consists of the predefined basic mean $\sigma_{x\,met}$ depended on maximal admissible speed of railway line, which represents the accuracy of control measurements of the spatial track position.

Table 4

Maximal admissible speed of railway line	Basic mean $\sigma_{x\,met}$ in mm		
	Two mutual abeam directions	Difference of deviations in two mutual abeam directions	In height
50 km/h and more	60	25	3
Less then 50 km/h	60	30	5

6. CONTRIBUTION

Experimental measurements of railway track confirm the convenience of using GPS method in railway geodesy especially, from the point of view of reached accuracy and time-consuming. However, GPS observations have shown, the necessity to combine this technique with classic surveying method in the localities with less satellite visibility.

Mathematical statistics provides many technologies to adjust local reference system, e.g. adjustment of free or fixed local networks, as it is described in (15). We decided to use combined adjustment procedure with using heterogeneous data, which predicts the variance-component estimation. Our proposal came out from the unifying the precision of the measured data into one data file, because the terrestrial measurements supplement the non-measurable values by GPS method.

In the case of mixed observations, the variance component estimation assumes to define a prior precision. Choice of adjustment procedure depends on the network configuration and software availability. Terrestrial measurements are supposed to involve into evaluation corrections depended on atmosphere characteristics or cartographic reference system, which affected measured data with systematic errors. Hence, a prior accuracy of GPS observations is not the subject of this study the variance-covariance matrices of GPS surveying were undertaken from post processing official software.

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