100 Years of Geodetic Measurements in the Piazza del Duomo (Pisa, Italy): Reference Systems, Data Comparability and Geotechnical Monitoring

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ABSTRACT

The entire Pisa plains is subjected to subsidence movements, including the world-renowned Piazza dei Miracoli. This phenomenon, as a whole, has long been investigated, but over time the value of the site has commanded specific and steady focus. Early geodetic observations, dating back to 1908-1912, included the monitoring of the inclination of the Torre Pendente (Leaning Tower). Only in the 1960s, geodetic and topographical measures were extended to the entire site, using as altimetric reference a benchmark, named φ', placed at the Baptistery. In 1989, a specific committee for the consolidation and restoration of the Leaning Tower was established, and in 1990 the Tower was closed to public access. The ensuing reconnaissance and executive planning provided the restoration and redesign of the levelling network for monitoring vertical movements of the site, also revealing that reference benchmark φ' was in fact subjected to subsidence. This issue was overcome by instating benchmark #999, which was anchored to the deep sands layer and acted as altimetric reference for any successive survey. Since 2012, monitoring of the vertical movements in the Piazza has been entrusted to ASTRO Laboratory of Pisa University. Lately, some interest has developed for investigating the evolution over time of subsidence in the site, thus requiring the use of a shared reference for different data sets.

These uncertainties have had relevant consequences on monitoring of monuments, in particular about the tower, which has been subjected to important stabilization measures during the last decade of XX century. Since the observational method has been applied to the tower, levelling surveys became mandatory; a thorough discussion about reference points has also been carried out.

The present paper has primarily historic connotations and describes the procedures followed to attune the diverse data sources in order to ensure their usability after roughly 100 years.

I. Introduction

The levelling surveys carried out during the past 100 years in the Piazza dei Miracoli have been targeted, until the 1990s, to the assessment and investigation of relative vertical shifts among the benchmarks laid out over the Piazza, rather than monitoring how subsidence, which involves the entire Pisa plains, is related with the Piazza and its reference benchmark.

The earliest surveys aimed at the analysis of vertical shifts among benchmarks date back to 1911. In that occasion, the reference benchmark, i.e. the one whose elevation is assumed constant between successive surveys, was provided by a line, signed by the initials "L.P.", etched on the right-hand jamb of the main door of the Baptistery. The benchmark was included in the National Geodetic Levelling Network of the Italian Geographical Military Institute (I.G.M.), and in the present paper will be referred to as IGM-CV-1886 (Figure 1).

The elevation of this benchmark has been defined as that measured by I.G.M. in 1886 at 6.7917m above mean sea level.

Explanatory notes accompanying the technical reports for the 1965-1966 (Istituto Geografico Militare, 1965: Istituto Geografico Militare, 1966) altimetric monitoring surveys state that "*l'interesse è rivolto ai confronti tra dislivelli e per questo scopo il riferimento altimetrico ha poca importanza*" ("the focus lies in comparing elevation differences, and to this purpose altimetrical references are of little interest"), but "*… al fine di legare i raffronti relativi alla realtà della situazione più probabile…*" ("in order to link relative comparisons to the actual, most probable situation") IGM-CV-1886 was preserved as altimetrical reference with the absolute elevation of 1886.

Levelling surveys performed in 1965 and 1966 allowed to link vertical benchmark IGM-CV-1886 to horizontal benchmark φ', set up in 1928 on the threshold of the main door of the Baptistery near its right-hand jamb, below IGM-CV-1886 (Figure 1). Elevation of φ' was measured at 4.28361m and was held as reference for later surveys.

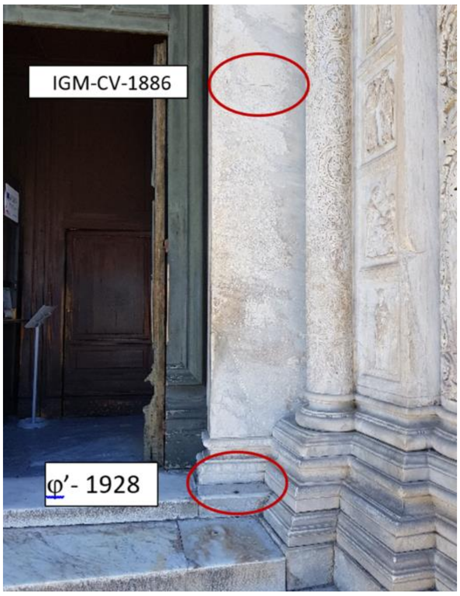


Figure 1. Location of IGM-CV-1886 and φ' benchmarks.

The statement contained in the explanatory notes is only partially shareable. The fact that level differences are the most important information is dependent on the continuity of the object structure. While the statement is true if referring to the surface on the entire Piazza, this is not the case when seeking information on the distortions which single building are subjected to, since level differences measured between different building include the effect of different concurring phenomena. As already discussed by Squeglia and Bentivoglio (Squeglia *et al.,* 2015), movements of the Piazza also definitely affect movements of the Tower, but referring to a benchmark placed on the Baptistery would also add interactions between Baptistery and soil of little or no relevance for the Tower to the level difference between Baptistery and Tower.

Also for this reason, in view of the works for the securing of the Tower in the 1990s, members of the International Committee for the safeguard of the Pisa Tower recognised the importance of assessing local subsidence and investigating its effects, along the ongoing monitoring of relative shifts. To this purpose, deep benchmark 999 was established in 1992 by anchoring a 60m invar rod to the deep sand layer (Figure 2); the following year, its elevation was measured at 3.57232m. Benchmark 999 and its elevation provide the current reference and constraint for all levelling surveys.

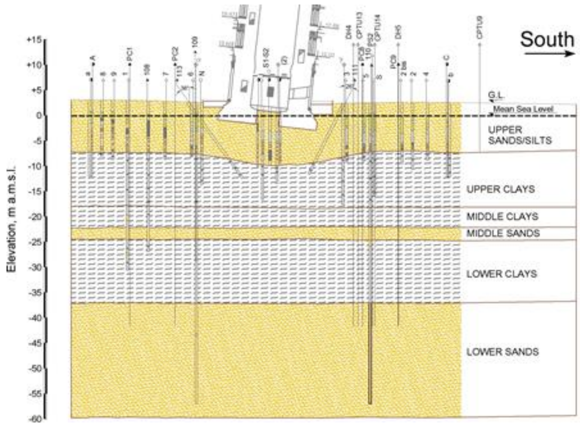


Figure 2. The subsoil of the Tower.

In recent years some interest arose in extending the investigations of subsidence in the Piazza area also prior to 1993. Achievement of this goal required tracking, from the late XIX century to date, of vertical shifts of both IGM-CV-1886 and φ'≡100 benchmarks, by researching historic data, and homogenization of elevation references.

II. MATERIALS

A. *Elevation monitoring from early 1900s to the 1990s*

Piazza dei Miracoli and its monuments (Figure 3) have long been subject to measurements. The main focus has consistently been placed on the Tower, whose inclination has always provided a topic for investigation and monitoring.

Earliest data related to inclination measures of the Tower come from E. Cresy and G. L. Taylor, which, in 1817, performed two overhang measures by dropping a plumb line from the VII order to the support plane of the elevation walls, resulting in 12 feet, 6 inches ¼ (3.82m) and 12 feet, 7 inches (3.84m), respectively (Lancellotta *et al.,* 2018; Lunardi*,* 1993).

Official levelling benchmarks set up in the Piazza dei Miracoli area date back to late XIX century.

From 1878 to 1900, I.G.M. set up and measured the foundation National Altimetric Network on the entire national territory.

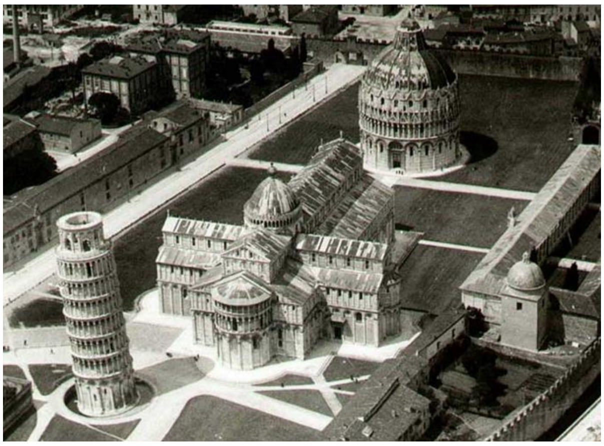


Figure 3. Pisa, Piazza dei Miracoli, 1931 (Historic archive, Family Gen. Enrico Pezzi)

Within these proceedings, in 1886 two benchmarks were set up by the main entrance of the Baptistery and subsequently measured (Regia Commissione Geodetica Italiana, 1911):

- A circle, etched on the last step by the left-hand jamb of the door facing the Cathedral, acting as horizontal benchmark with absolute elevation of 4.2930m. This benchmark, referred to as IGM-CO-1886 in the paper, was destroyed after 1943.
- Vertical benchmark IGM-CV-1886, made up by a line, captioned with the letters "L" and "P", etched on the right-hand jamb of the above mentioned door (Figure 1; Figure 4), with absolute elevation equal to 6.7917m.

Figure 4. Detail of benchmark IGM-CV-1886

Archive researches show that around the same time other benchmarks, set up on the floor of the christening font in the Baptistery and on the ground floor by the Tower entrance, were measured. These benchmarks have all been destroyed upon reconstruction works.

To the best of the Authors' knowledge, until 1908 no geometric levelling was provided to monitor vertical shifts of the Piazza and the Torre. In 1907, alarming news about Tower stability captured the attention of the competent authorities, which, between December 1907 and February 1908, established the "*Commissione per lo studio delle condizioni statiche del Campanile*", also known as Bernieri Committee (1911) (Commissione Pisana per gli Studi sulla Torre Pendente, 1927). The members of the Committee were: dr. Giovanni Cuppari (Hydraulic Engineer), Professor Paolo Pizzetti (Chair of Geodesy at Pisa University), Professor Mario Canavari (Chair of Geology at Pisa University), Dr. Francesco Bernieri (Head Engineer at the Pisa Municipality) and Dr. Agenore Socini (Head of the Pisa Bureau of Monuments).

The Committee presented the results of their investigations in several reports published by Opera Primaziale Pisana in 1913 (Commissione Pisana per gli Studi sulla Torre Pendente, 1913). The reports of Dr. Bernieri and Professor Pizzetti are most relevant for the purposes of the present paper.

In "Descrizione Generale e rilevamento del Campanile" dr. Bernieri provided building information, main dimensions and inclination measures, along with several drawings. Besides, by comparing " La pendenza attuale in confronto di quella d'altri tempi" (current vs. ancient inclination) with particular focus to the "piombatura" (plumblining) performed 1817 by Cresy and Taylor (Cresy *et al*., 1829), he demonstrated that until 1911 the inclination had increased by 5.5 millimeters for each meter of Tower axis.

Professor Pizzetti reported on "Studi Geodetici" (geodetical investigations), declaring that, around the Tower, 6 interred benchmarks – labelled with the letters A to F – had been set up in protective cockpits, providing for robustness and durability. For each benchmark, the reference was provided by the flat top of a bronze pin embedded in the foundations, on which the corresponding letter was etched, along with the wording "*Studi Campanile 1910-11*", Figures 5 and 6.

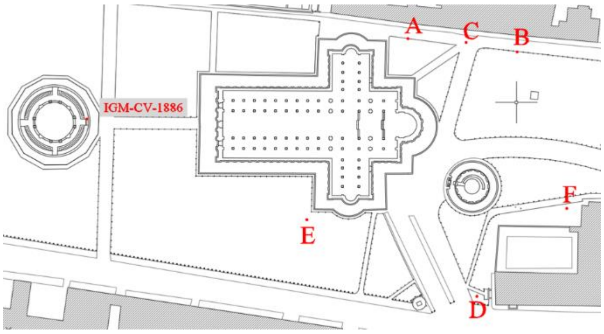


Figure 5. Layout of the 1911 benchmarks in Piazza dei Miracoli

Figure 6. Detail of a 1911 benchmark

These benchmarks served as both vertices of a small trigonometric network, providing framing support for the Tower overhang measures, and geometric levelling benchmarks of the historic levelling line for elevation surveys, along with benchmark IGM-CV-1886.

In 1911, Professor Pizzetti calculated the maximum Tower overhang, by comparing the horizontal angle measures of the right-hand tangents from benchmark E to rings I and VII. The measures were carried out on June 22nd and July 9th, with a calculated overhang of 2.899m between those rings.

The instrument used in these measurements was the 1896 Salmoiraghi Universale #19701 of the "Gabinetto di Topografia" of the Regia Università di Pisa (currently kept at the ASTRO Laboratory of the Pisa University), with 26cm graduated circles, direct 2" readings and graduation division by movable micrometer (Figure 7).

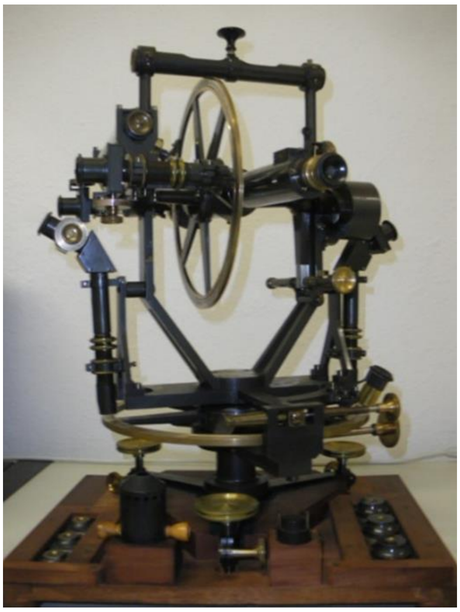


Figure 7. 1896 Universale Salmoiraghi n° 19701, used since 1911 for Bell Tower overhang measures

Also in 1911, Pizzetti performed the geometric levelling survey, based on the absolute elevation of benchmark IGM-CV-1886 with an unspecified instrument. Table 1 shows the elevation of the benchmarks set up by Pizzetti in 1911.

Table 1. Elevation of the Pizzetti benchmark as per the 1911 levelling survey

Benchmark	Elevation (m)
IGM-CV-1886 (reference)	6.7917
A	3, 601
B	3,509
C	3,491
D	3,156
E	3,297
F	3,199

In 1920, new measurements in the National Altimetric Network resulted in fixing the absolute elevation at 6.79170m to IGM-CV-1886, as shown in Table 2.

Table 2. Official absolute elevation of benchmark IGM-CV-1886 in 1920

Benchmark	Elevation (m)
IGM-CV-1886	6.79170

After 1911, overhang measurements were carried out on a regular basis by Professor Giovanni Cicconetti, following the methodology proposed by Pizzetti and using the same instrument. Archive research of any levelling surveys carried out in the same time span has been unsuccessful so far.

In 1918, 1923 and 1926, overhang measurements highlighted an ongoing increase of 9mm between rings I and VII. Based on this evidence, in 1927 Pisa Authorities instated a Technical Board, whose members included Professors Giovanni Cicconetti and Gino Cassinis (Commissione Pisana per gli Studi sulla Torre Pendente, 1927).

As regards topographic measurements, the 1927 Board suggested surveying and monitoring activities as follows:

- Survey of the Bell Tower.
- Systematic continuation of the optical overhang observations in the month of June of each year, according to the methodology of Pizzetti.
- Setup of six levelling benchmarks, made up by six bronze spikes embedded in the stone and featuring spherical cap heads. Four of these, labelled as α', β', γ', δ', were fixed at the base of Tower columns 1, 4, 7 and 11 (numbering clockwise from the left of the entrance). Two benchmarks were set up far away from the Tower: benchmark ψ' in the Archiepiscopal Palace, at the base of the first column on the right, and benchmark ϕ' at the threshold of the main door of the Baptistery by the right-hand jamb, below IGM-CV-1886.
- Precision geometric levelling twice a year, in June and December.

Figure 8 shows the location of the levelling benchmarks as of 1928.

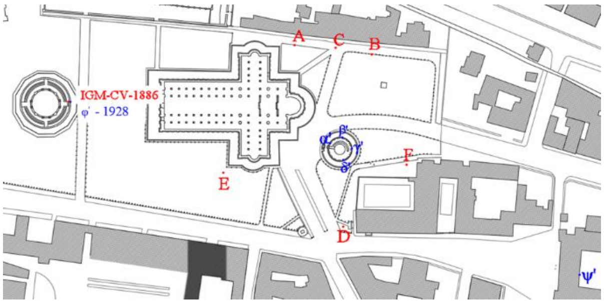


Figure 8. Levelling benchmarks as of 1928, showing in red those dating to 1911 and in blue those added in 1928

Between 1928 and 1929, the elevation of the six new benchmarks was calculated via geometric levelling linked to benchmark IGM-CV-1886, with the absolute elevation calculated in 1886. Also in this case, there is no evidence of the instruments used. Results of this levelling campaign are reported in Table 3.

Table 3. Elevation of 1928 benchmarks, as per the 1928 levelling survey

Benchmark	Elevation (m)
IGM-CV-1886 (reference)	6.7917
φ'	4.2836
α'	2.6217
β'	3.0078
γ'	2.3549
δ'	1.4521
ψ'	5.7881

Until 1965, overhang measures were carried out yearly, save some brief gaps, following the same methodology and using the same instrument.

The Authors have no evidence of elevation measures carried out in the same time span on the levelling benchmarks set up in the Piazza.

Archive documents, provided courtesy of I.G.M., show that in 1943 horizontal benchmark IGM-CO-1886, placed by the left-hand jamb of the main Baptistery door, was given absolute elevation of 4.3146m based on new measures in the National Altimetric Network. The same source provides no evidence as to benchmark IGM-CV-1886.

In the 1950s the levelling campaign of the new National Altimetric Network was launched, also including vertical benchmark IGM-CV-1886, with absolute elevation equal to 6.6756m (Table 4), while horizontal benchmark IGM-CO-1886 was no longer visible. Available documentation provides no evidence of the inclusion of benchmark φ' in the National Altimetric Network, and therefore its absolute elevation is unknown for the time span considered.

Table 4. Official absolute elevation of benchmark IGM-CV-1886 in 1951

Benchmark	Elevation (m)
IGM-CV-1886	6.6756

In 1964, the Ministry of Public Works established a new Committee for the consolidation of the Leaning Tower, also known as Polvani Committee after the name of its President. In 1965, the Committee entrusted I.G.M. with the task of carrying out monitoring surveys for both planimetry and elevation. To the purposes of this paper, only information relevant to levelling surveys is reported.

I.G.M. redesigned the geometric levelling network by splitting it in two independent and linked sections (Istituto Geografico Militare, 1965; Istituto Geografico Militare, 1966).

The first section was designed to monitor the base of the Tower, and provided the installation, at the foot of each column, of nickel-plated bronze bolts with spherical head, labelled 1 to 15; the four benchmarks set up in 1928 were also checked. Benchmarks α', γ', δ' were in good maintenance state, while β', no more visible, was reinstalled, keeping the original name. The base of the Tower eventually carried 19 benchmarks, 3 of which (α', γ', δ') dating back to 1928.

I.G.M. measured level differences between the benchmarks set up in 1928 and those set up at the corresponding columns in 1965, allow to calculate the elevation of the 1928 benchmarks based on those set up in 1965.

The second section of the network was designed to extend the investigations on elevation shifts to both the immediate surroundings of the Tower and the entire Piazza dei Miracoli. To this purpose, new benchmarks were laid out over the involved area, so to integrate both the 1911 and 1928 historic benchmarks. The six (A, B, C, D, E, F) benchmarks set up by the Bernieri Committee were checked and respectively renamed as A_b, B_b, C_b, D_b, E_b, F_b. Since these flat top benchmarks were unsuited to the instruments used at the time, six stainless, spherical cap steel bolts were embedded on the same support plates close by the old benchmarks. The new benchmarks were named A, B, C, D, E, F accordingly to their old reference (Figure 6). Level differences with the old benchmarks were measured, allowing to calculate elevation of the latter based on the new data. Checks of benchmarks ψ', at the base of the first column on the right in the Archiepiscopal Palace courtyard, and φ', on the threshold of the main door of the Baptistery by the right-hand jamb (below IGM-CV-1886), both set up in 1928, revealed good maintenance conditions and steadiness.

The levelling network established in 1965 by I.G.M. included, either directly or indirectly, eleven historic benchmarks set up in 1911 and 1928 (Table 5).

Geometric levelling measures used a Zeiss A level (ID #62160) fitted with a parallel plane plate micrometer, and flat-heeled levelling rods with half-centimeter divisions on invar tape. The elevation reference was provided by benchmark IGM-CV-1886 with absolute elevation at 1886. The elevation survey allowed to link directly horizontal benchmark φ', by the Baptistery door, with vertical benchmark IGM-CV-1886, resulting in a level difference of -2.50809m.

The elevation of φ' in 1965 was fixed at 4.28361m. This benchmark, with the associated elevation value, was used as elevation reference until 1993. Anyway, it is worth noting that its elevation was determined based on the 1886 absolute elevation of vertical benchmark IGM-CV-1886.

Until 1993, elevation monitoring of the benchmarks set up in the Piazza and Tower areas focused on checking their relative shifts rather than checking and investigating local effect of subsidence.

Useful information in regard to subsidence in the Pisa plains are provided by the investigations carried out by Palla and Geri (Palla *et al.*, 1976; Geri *et al.*, 1985).

These Authors stated that the entire Pisa plains, due to the subsoil structure, is subjected to subsidence phenomena. In particular, they pointed out that, relative to a bedrock benchmark, the reference benchmark IGM-CV-1886 at the Baptistery has sustained, between 1886 and 1981, vertical shifts showing alternating trends, as follows:

- Between 1951 and 1975, the benchmark shift was assessed at -37mm.
- Between 1975 and 1981, the benchmark shift was assessed at +6mm.

One of the main causes of the trend reversal shown by the shifts in the 1951-1975 period lies in the lowering of the water table due to the intense withdrawals and pumping of deep water carried out after the war. Based on this evidence, in 1973 the withdrawal of deep water in the surroundings of the Tower was banned, with positive consequences materialized in the trend reversal in the 1975-1981 period.

Table 5. Historic benchmarks

1993 network		1965 network		Historic benchmarks		Link between 1965 benchmark and corresponding historic benchmark	Transition level difference - 1965 to historic benchmark
Benchmark ID	Set up year	Benchmark ID	Set up year	Linked historic benchmark ID	Set up year		
1010	1965	A	1965	A_b	1911	Indirect	-0.02201
1008	1965	B	1965	B_b	1911	Indirect	-0.02203
1009	1965	C	1965	C_b	1911	Indirect	-0.02055
1013	1965	D	1965	D_b	1911	Indirect	-0.02218
1011	1965	E	1965	E_b	1911	Indirect	-0.02138
1018	1965	F	1965	F_b	1911	Indirect	-0.02195
901	1965	1	1965	α′	1928	Indirect	-0.00927
907	1965	7	1965	γ′	1928	Indirect	-0.08324
911	1965	11	1965	δ′	1928	Indirect	-0.01788
1020	1928	ψ′	1928	ψ′	1928	Direct	-
100	1928	φ′	1928	φ′	1928	Direct	-

It is undisputed that the subsidence has affected the Baptistery benchmark and, as a consequence, any other linked benchmarks in the Piazza.

B. *Monitoring of elevation since the 1990s*

Starting in the 1970s, the inclination of the Tower has sustained a marked increase, leading to present danger of collapse. In 1989, the Ministery of Public Works appointed the "Comitato internazionale per la salvaguardia della Torre di Pisa" which, considering the critical situation of the monument, recommended to ban public access. On January 7th, 1990 the mayor of Pisa enforced the ban, which was to be lifted December 15th, 2001, upon completion of the consolidation and restoration works.

In 1991, upon completion of new measure sets of the National Levelling Network, absolute elevations of benchmarks IGM-CV-1886 and φ′ were updated, as reported in Table 6.

In the same year, the Committee entrusted the Dipartimento di Ingegneria Idraulica, Ambientale e del Rilevamento (DIIAR – Department of Hydraulics, Environment and Surveying Engineering) of the Milan Polytechnic to carry out geodetic and topographic surveys in the Piazza and Tower areas, as part of the reconnaissance –executive planning and intervention operations on the Tower.

Table 6. Official absolute elevations of benchmarks IGM-CV-1886 and φ′ in 1991

Benchmark	Elevation (m)
IGM-CV-1886	6.6033
φ′	4.0951

The Milan Polytechnic supervised levelling surveys for elevation monitoring until 2010, retaining the same layout of high-precision levelling network used in the past. Existing benchmarks were checked and replaced if needed, and new benchmarks were set up to improve the definition of elevation variations in the Piazza and foremost in the Tower areas. Besides, all benchmarks were renamed. Table 5 reports historic benchmarks directly or indirectly included in the 1993 network with the relevant nomenclature.

The scheduled works for Tower safety and consolidation required materialization of a reference benchmark, so to provide steadiness over time and avoid any shift due to local or surface disturbances. In fact, it was clear that subsidence of the whole Piazza also affected Baptistery benchmarks IGM-CV-1886 and φ′≡100. Given their instability over time, comparisons between compensated heights in different campaigns could include unwanted systematic effects, providing incorrect readings of the absolute subsidence phenomenon. For this reason, in 1992 a new deep elevation reference, named 999, was set up and linked to the lower sand horizon.

On May 29th, 1993 the first geometric levelling measure of the new benchmark network was carried out, to define both the "zero" situation prior to the set-up of the lead counterweights on the Tower base, and the elevation of the new elevation reference benchmark 999, relative to the previous reference benchmark φ′≡100 at the Baptistery.

Levelling operations were carried out using a Zeiss Ni1 automatic level fitted with a half-centimeter parallel plane plate micrometer, yielding a mean square error equal to ± 0.2mm/km, and a pair of 3m levelling rods with half-centimeter division on invar tape, fitted with telescopic supports and spherical bubble for vertical standing. The compensation of the entire levelling network assumed benchmark φ′≡100 as elevation reference, with elevation equal to 4.28361m as defined by the geometric levelling campaign of 1965.

The compensation allowed to determine and fix the elevation of benchmark 999 at 3.57232m, which acted as reference for any future levelling operation. The establishment and use of new reference benchmark 999 enabled to highlight both movements of the Baptistery benchmark φ′≡100 relative to benchmark 999 and movements of the Piazza benchmarks without the influence of the very subsidence of φ′≡100.

In the 2000s DIIAR replaced the levelling instrumentation described above with a Leica DNA automatic digital level and invar barcode levelling rods. Available documentation does not specify neither exact time nor technical specifications.

In 2006 the National Levelling Network was updated with new measures, which resulted in reassigning absolute elevations to benchmarks IGM-CV-1886 and φ'≡100, as reported in Table 7.

Table 7. Official 2006 absolute elevations of benchmarks IGM-CV-1886 and φ'≡100

Benchmark	Elevation (m)
IGM-CV-1886	6.5818
φ'≡100	4.0756

In 2012, the ASTRO Laboratory of Pisa University has been appointed the responsibility for levelling surveys. Instrumentation used for the task included a digital Leica DNA03 automatic level, providing a mean square error equal to ± 0.3mm/km, and two barcoded invar levelling rods (Leica GPCL2 and GPCL3), with spherical bubble and telescopic supports for vertical standing.

Table 8 reports the elevation of benchmark φ'≡100 as determined by surveys from 1993 through 2016. The altimetric reference is provided by benchmark 999, with an elevation of 3.57232m, as per the May 29th, 1993 levelling survey.

Table 8. Elevation of benchmark φ'≡100 as determined by surveys from 1993 through 2016

Survey supervision	Date	Elevation of benchmark φ'≡100 (m)	Elevation of benchmark 999 (m)
POLIMI-DIIAR	29-05-93	4.28361	3.57232
POLIMI-DIIAR	21-08-93	4.28239	3.57232
POLIMI-DIIAR	21-02-94	4.28336	3.57232
POLIMI-DIIAR	06-02-95	4.28266	3.57232
POLIMI-DIIAR	26-02-96	4.28266	3.57232
POLIMI-DIIAR	30-11-96	4.28205	3.57232
POLIMI-DIIAR	04-05-98	4.28178	3.57232
POLIMI-DIIAR	26-07-99	4.27930	3.57232
POLIMI-DIIAR	10-10-00	4.27838	3.57232
POLIMI-DIIAR	08-11-01	4.27780	3.57232
POLIMI-DIIAR	29-10-03	4.27524	3.57232
POLIMI-DIIAR	21-10-04	4.27526	3.57232
POLIMI-DIIAR	11-07-05	4.27511	3.57232
POLIMI-DIIAR	26-06-06	4.27410	3.57232
POLIMI-DIIAR	07-07-08	4.27291	3.57232
POLIMI-DIIAR	13-07-10	4.27243	3.57232
UNIPI-ASTRO	29-06-12	4.27089	3.57232
UNIPI-ASTRO	30-06-14	4.27029	3.57232
UNIPI-ASTRO	30-06-16	4.26988	3.57232

III. METHODS

Earliest data about the official absolute elevation of benchmark φ'≡100 date back to 1991 (Table 6). In order to reconstruct its historic series of altimetric data (prior to 1991) the level difference relative to vertical benchmark IGM-CV-1886, $\Delta^{YYYY}_{\varphi',\mathrm{IGM-CV-1886}}$, has been assumed equal to 2.5081m. This assumption is backed by the fact that this value is exactly the level difference between these two benchmarks in measures of both 1928 and 1965, rounded to 10^{-4}m.

Based on this assumption, the elevation of benchmark prior to 1991 have been calculated relative to the official absolute elevations of vertical benchmark IGM-CV-1886 for years 1920 (Table 2) and 1951 (Table 4), applying the formula:

$$Q^{YYYY}_{\varphi'(bbbb)} = Q^{YYYY}_{\mathrm{GM-CV-1886}(bbbb)} - \Delta^{YYYY}_{\varphi',\mathrm{IGM-CV-1886}} \quad (1)$$

The symbols used stand for:

- $\Delta^{YYYY}_{NN,NN}$, level difference;
- $Q^{YYYY}_{\mathrm{NN}(bbbb)}$, absolute elevation;
- $YYYY$, year of survey from which level difference or elevation have been derived;
- NN, benchmark ID;
- $bbbb$, year of assignment/update of the altimetric reference associated with the elevation.

The levelling survey of May 29th, 1993 allowed to calculate the absolute elevation of benchmark 999, based on the assumption that the elevation of benchmark φ'≡100 was the 1991 official elevation, as shown in Table 6.

Calculation of this elevation provides determination of level difference $\Delta^{1993}_{\varphi',999}$ between benchmark φ'≡100 and newly instated altimetric reference benchmark 999 from the compensated elevation as per the May 29th, 1993 levelling survey:

$$\Delta^{1993}_{\varphi',999} = Q^{1993}_{999(1886)} - Q^{1886}_{\varphi'(1886)} \quad (2)$$

Absolute elevation of benchmark 999 in 1993, $Q^{1993}_{999(1991)}$, is calculated by adding to level difference $\Delta^{1993}_{\varphi',999}$, previously calculated, the official absolute elevation of benchmark φ'≡100 for year 1991, $Q^{1991}_{\varphi'(1991)}$, as reported in Table 6:

$$Q^{1993}_{999(1991)} = \Delta^{1993}_{\varphi',999} + Q^{1991}_{\varphi'(1991)} \quad (3)$$

The last official data of absolute elevation of benchmark φ'≡100 dates back to 2006 and is reported in Table 7. In the same year, DIIAR completed a geometric levelling survey on the benchmarks of the altimetric monitoring network on the entire Piazza, whose results are shown in Table 8. Elevation of benchmark φ'≡100 was fixed at 4.27410m relative to the altimetric reference provided by benchmark 999.

The concurrence of altimetric data for 2006 allows to calculate absolute elevations for this epoch of both benchmark 999 in the national altimetric reference system for 2006, and benchmark 100 in national altimetric reference system from 1993 through 2016.

Calculation of absolute elevation $Q^{2006}_{999(2006)}$ of benchmark 999 in the national altimetric reference system for epoch 2006 provides to determine the level difference $\Delta^{2006}_{\varphi',999}$ between benchmark φ'≡100 and newly instated altimetric reference benchmark 999 from compensated elevations derived from the 2006 levelling survey:

$$\Delta^{2006}_{\varphi',999} = Q^{1993}_{999_{(1886)}} - Q^{2006}_{\varphi'_{(2006)}} \quad (4)$$

Absolute elevation $Q^{2006}_{999_{(2006)}}$ of benchmark 999 in the national altimetric reference system for epoch 2006 was calculated by adding level difference $\Delta^{2006}_{\varphi',999}$ to the official absolute elevation of benchmark φ'≡100 for the same epoch:

$$Q^{2006}_{999_{(2006)}} = Q^{2006}_{\varphi'_{(2006)}} + \Delta^{2006}_{\varphi',999} \quad (5)$$

Calculation of absolute elevations of benchmark φ'≡100 from 1993 to 2016, $Q^{YYYY}_{\varphi'_{(2006)}}$, in the national altimetric reference system for epoch 2006 required previous calculation of level difference $\Delta^{YYYY}_{999,\ \varphi'}$ between benchmarks 999 and φ'≡100 for each year in which levelling surveys had been carried out.

Once these level differences have been calculated, absolute elevation $Q^{YYYY}_{\varphi'_{(2006)}}$ of benchmark φ'≡100, in the national altimetric reference system for epoch 2006, for every levelling year, have been calculated according to the following relation:

$$Q^{YYYY}_{\varphi'_{(2006)}} = Q^{2006}_{999_{(2006)}} + \Delta^{YYYY}_{999,\ \varphi'} \quad (6)$$

IV. Results

Absolute elevation of benchmark φ'≡100 for years 1928 and 1951 has been calculated using formula 1. Table 9 shows the results.

Table 9. Absolute elevation of benchmark φ'≡100 prior to 1991, relative to the official absolute elevation of benchmark IGM-CV-1886 for years 1928 and 1951

Year	Elevation (m)
$Q^{1928}_{\varphi'_{(1920)}}$	4.2836
$Q^{1951}_{\varphi'_{(1951)}}$	4.16753

Level difference $\Delta^{1993}_{\varphi',999}$ between benchmark φ'≡100 and newly instated altimetric reference benchmark 999, using compensated elevations as per the May 29th, 1993 levelling survey, has been calculated applying formula 2, and was equal to:

$$\Delta^{1993}_{\varphi',999} = 3.57232 - 4.28361 = -0.71129\ m \quad (7)$$

Absolute elevation of benchmark 999 in 1993, $Q^{1993}_{999_{(1991)}}$, calculated with formula 3, was equal to:

$$Q^{1993}_{999_{(1991)}} = -0.71129 + 4.0951 = 3.38381m \quad (8)$$

Level difference $\Delta^{2006}_{\varphi',999}$ between benchmark φ'≡100 and newly instated altimetric reference benchmark 999, using compensated elevations as the 2006 levelling survey, has been calculated applying formula 4, and was equal to:

$$\Delta^{2006}_{\varphi',999} = 3.57232 - 4.27410 = -0.70178\ m \quad (9)$$

Absolute elevation $Q^{2006}_{999_{(2006)}}$ of benchmark 999 in the national altimetric reference system at epoch 2006, calculated applying formula 5, was fixed at:

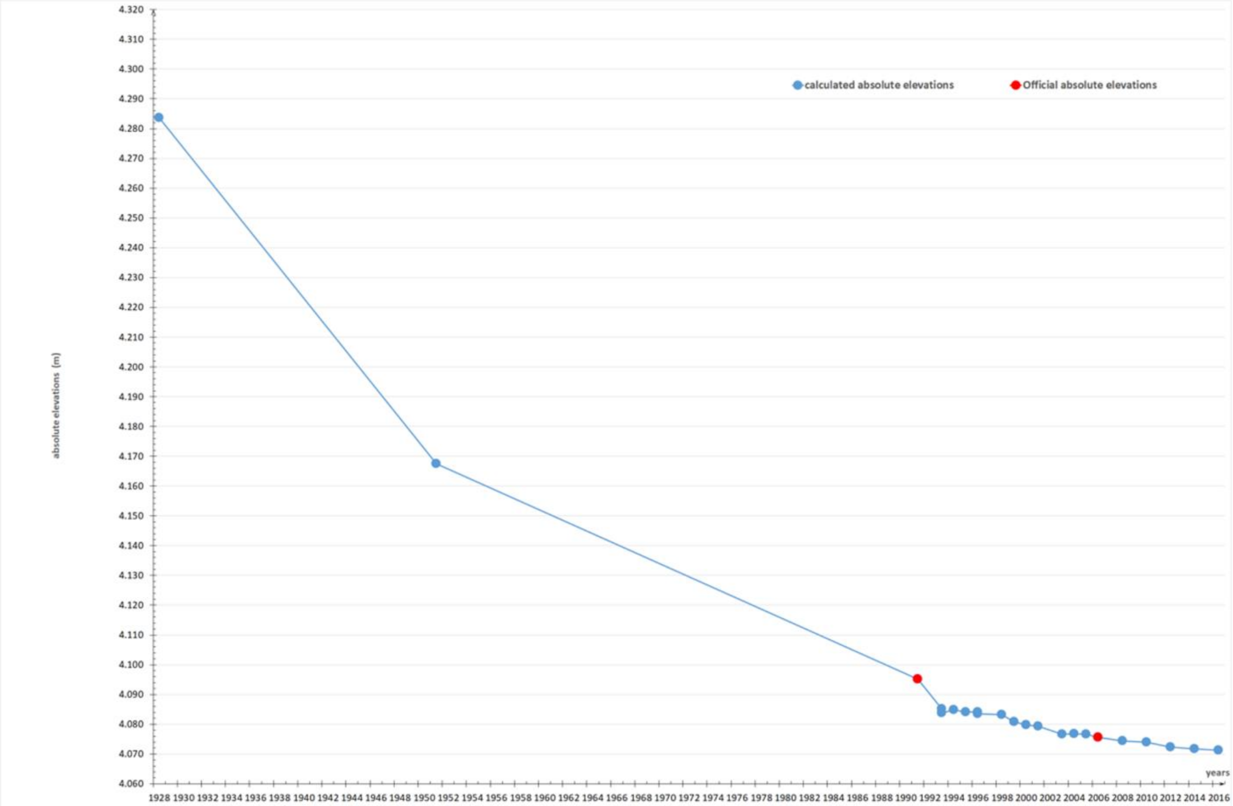
$$Q^{2006}_{999_{(2006)}} = 4.0756 - 0.70178 = 3.37382\ m \quad (10)$$

Absolute elevation $Q^{YYYY}_{\varphi'}$ of benchmark φ'≡100 in the national altimetric reference system at epoch 2006, was calculated for levelling years 1993 to 2016 applying formula 6. Results are shown in Table 10.

Table 10. Absolute elevations (1993 – 2016) of benchmark φ'≡100 in the national altimetric reference system for epoch 2006

Survey supervisor	Date	Elevation (m)
POLIMI-DIIAR	29-05-93	4.08511
POLIMI-DIIAR	21-08-93	4.08389
POLIMI-DIIAR	21-02-94	4.08486
POLIMI-DIIAR	06-02-95	4.08416
POLIMI-DIIAR	26-02-96	4.08416
POLIMI-DIIAR	30-11-96	4.08355
POLIMI-DIIAR	04-05-98	4.08328
POLIMI-DIIAR	26-07-99	4.08080
POLIMI-DIIAR	10-10-00	4.07988
POLIMI-DIIAR	08-11-01	4.07930
POLIMI-DIIAR	29-10-03	4.07674
POLIMI-DIIAR	21-10-04	4.07676
POLIMI-DIIAR	11-07-05	4.07661
POLIMI-DIIAR	26-06-06	4.07560
POLIMI-DIIAR	07-07-08	4.07441
POLIMI-DIIAR	13-07-10	4.07393
UNIPI-ASTRO	29-06-12	4.07239
UNIPI-ASTRO	30-06-14	4.07179
UNIPI-ASTRO	30-06-16	4.07138

These results are presented in Figure 9.



	1928	1951	1991	1993	1993	1994	1995	1996	1996	1998	1999	2000	2001	2003	2004	2005	2006	2006	2008	2010	2012	2014	2016
calculated absolute elevations	4.2836	4.16753	4.09510	4.08511	4.08389	4.08486	4.08416	4.08416	4.08355	4.08328	4.08080	4.07988	4.07930	4.07674	4.07676	4.07661	4.0756	4.07560	4.07441	4.07393	4.07239	4.07179	4.07138
Official absolute elevations			4.09510														4.0756						

Figure 9. Elevation trend for benchmark φ’≡100

Figure 9 shows the elevation trend for benchmark φ’≡100 as reconstructed according to the methods illustrated above. The resulting curve is typical of a subsidence phenomenon as the consequence of primary consolidation of clayey soil layers, most probably correlated with the lowering of the water level in the lower sands. Figure 10 shows the time trend of water level in sand layers starting from 1966, highlighting consistently lower values compared with mean sea level, which in turn triggers compacting of clayey layers. The relevant thickness of the latter results in an extremely slow and surely ongoing consolidation process, as highlighted by values for the last 20 years.

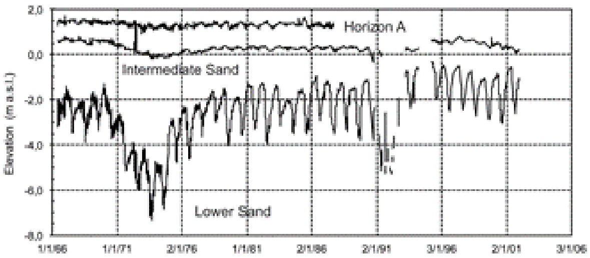


Figure 10. Trend of water level in sand layers starting from 1966

The comparison of absolute elevation of benchmark 999 in 1993 (epoch 1991), $Q^{1993}_{999_{(1991)}}$, and 2006 (epoch 2006), $Q^{2006}_{999_{(2006)}}$, shows that the benchmark has been subjected to a negative movement of almost one centimeter. Barring as a possible cause of this behaviour the uncertainty of the official absolute elevation, due for instance to recalculation over time of the National Altimetric Network, the phenomenon could be ascribed to water level reduction.

The presence of clayey layers underneath the lower sands has already been reported (Squeglia *et al*., 2015). For this reason, benchmark 999 – anchored to the deep sands – is affected by the deformations originating in the underlying layers, due to the consolidation triggered by the lowering of the mean water level, which is as a consequence ultimately responsible for the negative movement of benchmark 999.

By analysing the graphics of Figure 9, vertical movements of benchmark φ’≡100 for the period 1951-1975 has been calculated using linear interpolation at -43mm. This result substantially matches the findings of Palla and Geri (Palla *et al.*, 1976; Geri *et al*., 1985), who fixed this value at -37mm. A possible explanation for this 6mm difference could lie in the fact that those Authors used as altimetric reference the elevation of a benchmark materialized on the Monte Pisano bedrock, as such subjected to the vertical movements of the Mid-Tuscany Ridge (Palla *et al.*, 1985).

V. Conclusions

The present paper reports for the first time a summary of topographical measures performed over more than 100 years in the Piazza dei Miracoli area. In particular, tracing back the elevation trend over time for some benchmarks in the Piazza demonstrated the usefulness of setting up reference benchmark 999, which can be assumed as external relative to points set up on monuments or on ground. Benchmark 999 is anyway subjected to subsidence due to changes in subsoil hydraulics.

Reconstruction of vertical movements of benchmark φ'≡100 over time allowed to define major affecting factors, along with their share of the total effect.

This step is fundamental for a correct definition of the movements, and therefore of the distortions, of each monument, in relation with their soil-monument interactions, in order to provide their correct preservation.

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