

Patterns of Greek ancient theatres

Generally, the typical pattern of the ancient theatres is symmetric, consisting of an arcuate wing (arc angle $140^\circ - 200^\circ$)

Ancient Theatre of Epidaurus

Makyneia Theatre

The Makyneia theatre is an atypical, asymmetric ancient theatre consisting of an arcuate and a linear wing

The ancient theatre of Makyneia

Makyneia theatre is at a distance 20 km north of Patras
It is dated to the 4th B.C.

The theatre is curved to a hill, and seats are formed with hewn sandstone slabs

It includes an arcuate wing, consisting 14 rows,
and a linear wing, consisting of 3 blocks

Probably the linear wing corresponded to the seats for honored people

Linear wing

The geometric pattern of the theatre

Due to the:

1. instability of the slabs (seats) foundation and
2. the bad geological conditions

many slabs are displaced from their initial position
Thus, the details of the original geometry of the structure are not known

In order to identify the pattern of the theatre a topographic study was made

Linear wing

Arcuate wing

Identification of the geometric structure of the theatre

Graphic technique

At a first step the identification of the theatre was based on graphic techniques (circles and straight lines drawn on transparent paper and superimposed to the plan of the theatre)

The graphic technique revealed that:

- the arcuate rows were constructed on the basis of concentric circles and
- the linear wing is perpendicular to the chord of the first arcuate row

In this study

In order to improve and confirm the modeling of the structure an analytical approach (LSQR method) was adopted

Analytical Technique

Our analysis was focused on the 12 lower, best preserved, rows of seats
In total, 183 slabs (hewn blocks of seats) were included in the analysis

The assumption of the analysis was:

Rows of seats correspond to concentric circles
All rows of seat except for the first are equidistant

Input data

1. the mean coordinates of the free edge of each block
2. the approximated radius of each row and
3. the approximate coordinates of the circle center

Modeling of the arcuate wing

The distance between an observation point $P(x_{ij}, y_{ij})$ and the circle center (x_o, y_o) is:

$$(x_{ij} - x_o)^2 + (y_{ij} - y_o)^2 = R_i^2$$

Also the radius R_i of the i th row of seats is given by the expression:

$$R_i = R_1 + c + i \times d$$

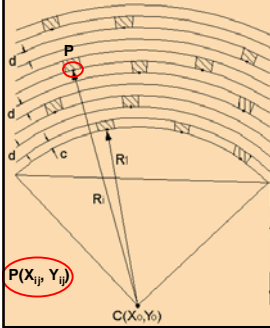
R_1 is the radius of the first row

c is the spacing between the first two rows

d is the spacing between the rest of the rows

Hence, the distance between each slab and the circle center is:

$$(x_{ij} - x_o)^2 + (y_{ij} - y_o)^2 = (R_1 + c + i \times d)^2$$



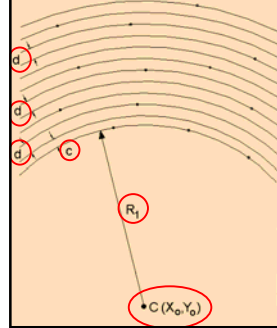
The observation equation was transformed into a linear equation using Taylor Series

a system of **183 linear equations** with **5 unknowns** was formed

The five unknowns were:

1. The circle center coordinates $C(x_o, y_o)$
2. The first row radius R_1
3. The spacing c between the first two rows
4. The spacing d between the rest of the rows

The system was solved using standard Least Squares techniques



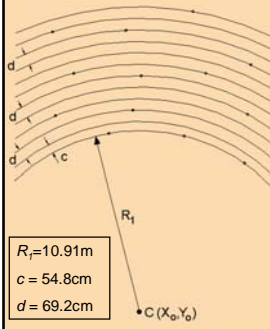
Results of Analytical and Graphic Technique

Estimates of the unknown parameters of design of the theatre (center of design, radii of circles, spacing of rows of seats) obtained by both approaches are very similar

The unknowns parameters were estimated with an accuracy:

- 3cm the radius of the lower row
- 2.5cm for the coordinates of the circle center
- 1cm for the spacing between the rows of seats

The high accuracy of the results is due to the high redundancy of the system



$R_1 = 10.91\text{m}$
 $c = 54.8\text{cm}$
 $d = 69.2\text{cm}$

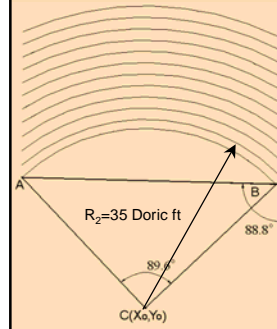
Geometric characteristics of the theater

Based on analytical calculation:

1. The lower row arc corresponds to an angle of 89.6°
2. The mean direction of the linear wing and the first arcuate row chord form a 88.8° arc

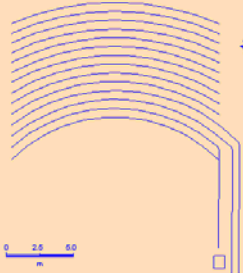
Within the errors of the analysis both these angles correspond to 90°

The radius of the second row corresponds to 35 Doric feet
(1 Doric foot = 0.327m)



Conclusions

- ☑ The plan of the Makyneia theatre was based on a 90° circle arc
- ☑ The linear wing is perpendicular to the chord of the circular wing
- ☑ The design of the theatre was based on exactly 35 Doric feet, a measure of distances in antiquity



- The analytical approach confirmed the results of a graphical approach
- Makyneia theatre was a small, provincial and unusual theatre, but it was constructed on the basis of a strict, geometric plan