Modelling an Ancient Theatre Using Analytical Techniques

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Key words: Makyneia, ancient theatre, modelling, analytical technique

SUMMARY

An ancient (4^{th} century BC) Greek theatre of a very unusual style has been excavated near Makyneia, close to Patras, Greece. This theatre consists of 14 arcuate rows of seats and a linear wing for prominent citizens (proedria). Because of the instability of the foundations of the slabs composing the rows of seats the reconstruction of the geometry of the structure is not easy. Until now a graphic method has been used in order to define the initial plan of the theatre. In this paper a analytical method for the reconstruction of the geometry of the theatre is presented.

In our study we approximated the curved rows of seats with circle arcs and by using LSQR we estimated five unknown parameters: the two coordinates of the circle center, the radius of the lower arc row of seats, the spacing between the two lower successive rows of seats and the spacing between successive upper rows. The estimated standard errors of the five parameters were between 1mm and 3 cm, due to the high redundancy of the observations (178). It was also estimated that the arc of the circle corresponding to the lower circular row of seats and the angle between the chord of this row and the rectilinear row are approximately 90^o. Furthermore, the radius of the circle of the second (design) row of seats corresponds to 35 Doric feet, a measure of length in antiquity.

Our analytical approach indicate that the ancient theatre was originally constructed on the basis of a 90^o circle arc, and its curved part is very similar to the other typical theatres of the period, differing only as far as the arc angle (90^o instead of 140^o-200^o) and the additional linear wing are concerned.

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1. INTRODUCTION

Archeological excavations brought to light, between 1984 and 1989, remains of a small ancient theatre next to the fortified archaeological site (Fig. 1), close to the modern village of Makyneia, at a distance approximately 20 km from Patras (Kolonas 1992/93).

The Makyneia theatre (4th century BC) is very unusual (Fig. 2; Frederiksen, 2000). While all the other known theatres of this period are symmetric (Dinsmoor, 1975), the Makyneia theatre is asymmetric and includes an arcuate wing, consisting of 14 rows of seats, and a linear wing at the right side, consisting of 3 rows of blocks, probably corresponding to seats for honored people. The seats were made of hewn sandstones slab, fixed on slightly excavated ground without mortar. Because of the instability of slab foundation and the bad geological conditions, many of these slabs are tilted or even have slid from their initial position. For this reason the details of the original geometry of the ancient structure are not known.

In 1998, a topographic study of the theatre (Fig.3) and the broader ancient site was made in the framework of the research activities of the Geodesy and Geodetic Applications Laboratory of the Department of Civil Engineering of the Patras University (Kolonas, 1998). An effort at the identification of the geometry of the ancient structure was made based on a graphic approximation of the ancient structure by idealized geometric figures (circles and straight lines; Palyvou, 2001). In order to improve the modelling of the structure, an analytical approach (LSQR method) was adopted in this article.

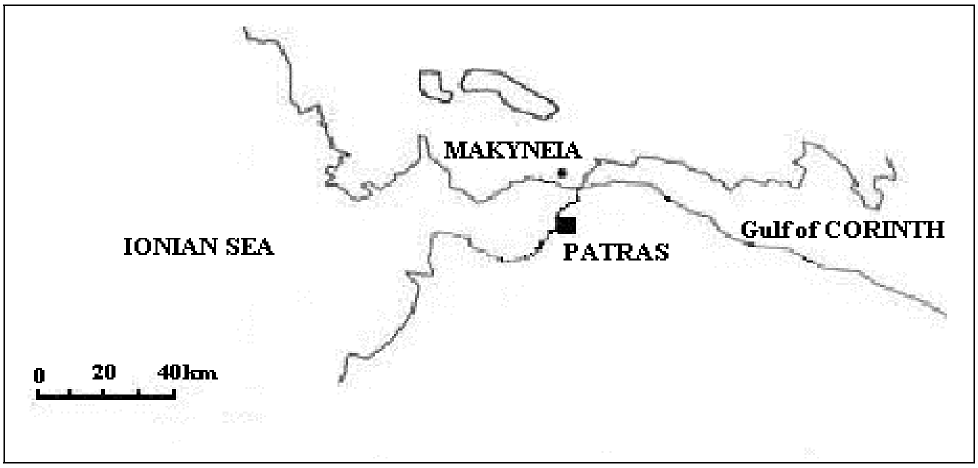

Figure 1: Location map of the ancient theatre Makyneia

Figure 2: Photo of the Makyneia theatre. The linear wing is to the right

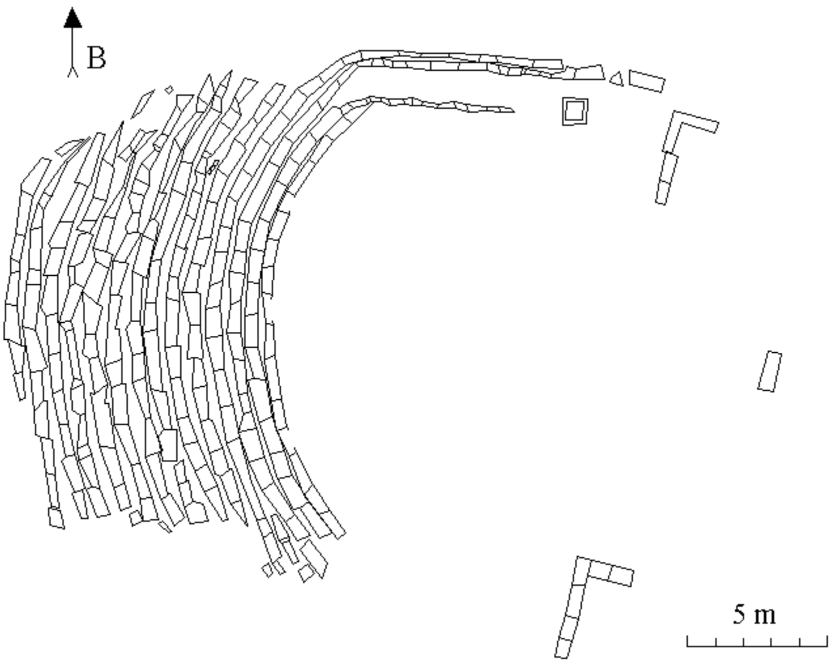

Figure 3: Plan of the remains of the ancient theatre of Makyneia, after Kolonas (1998), modified.

2. METHODOLOGY

The graphical technique revealed that the theatre consisted of circular concentric rows of seats and a row of seats consisting of three lines of hewn blocks. In order to improve and confirm this modelling, we approximated the rows of seats by concentric, equidistant circles and by straight lines using an analytical technique. Input in our analysis were the mean coordinates of the free edge of each slab apparently *in situ*. For each point a 2[nd] order equation which expressed the distance between the circle center and the point was defined. The system of these equations was solved for five unknown parameters (two circle center coordinates, first row radius, spacing between the two lower rows and spacing of all upper rows; Fig. 4) and their standard errors were computed with the least square method. The geometric characterizations of the theatre were subsequently defined on the basis of the adjusted data.

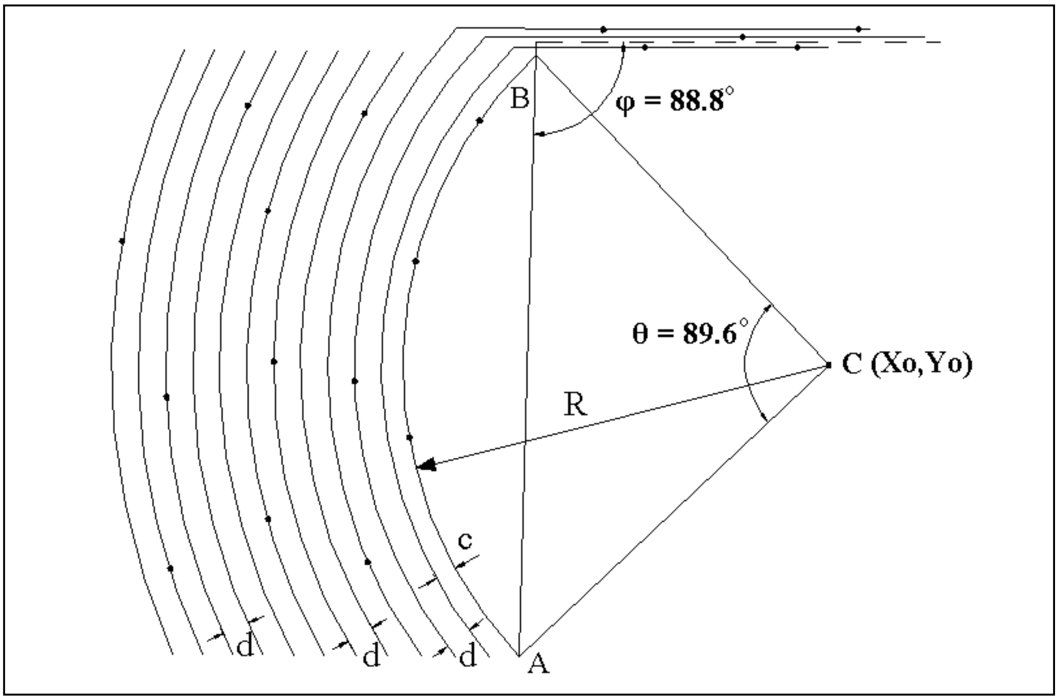

Figure 4: A sketch of the theatre design based on the parameters of our study. Dots indicate selected coordinates of slabs input in our study

3. MODELLING OF THE ARCUATE WING

Our analysis focused on the 12 lower, best preserved, arcuate rows of seats, excluding the upper two rows in which most of the slabs were displaced from their initial position. Some of the slabs of the 12 lower rows were also excluded, due to their poor preservation or dislocation. Finally the coordinates of 183 slabs were included for the analysis using an arbitrary coordinate system. Based on the coordinates of each slab and the approximated coordinates of the circle center, estimated from the graphical technique, the equation of radius (distance) was defined as:

$$(x_{ij} - x_0)^2 + (y_{ij} - y_0)^2 = R_i^2 \quad (1)$$

where x_{ij}, y_{ij} (i corresponds to the row number and the j corresponds to the slab number of the row) are the coordinates of the slab, x_o, y_o are the approximated coordinates of the circle center and R_i is the distance of these two points (R_i corresponds to the radius of the i row). The distance of each slab from the circle center was defined as a function of the radius of the lower arcuate row and of the spacing of the theatre rows of seats. The spacing of the two lower rows seemed to be smaller than of that of the higher rows. Consequently the radius of a row i was estimated from the expression:

$$R_i = R_1 + c + i \times d \text{ , i = 0, 1, …, 12} \quad (2)$$

where c is the spacing between the two lower arcuate rows and d the spacing between the upper rows (Fig. 4).

Taylor series were used in order to transform equation (1) into the linear equation (3):

$$\frac{x_{ij} - x_0}{R_i} \times \delta x + \frac{y_{ij} - y_0}{R_i} \times \delta y - R_1 - c - n \times d = -R_i \quad (3)$$

using equation (2). Table 1 shows slabs used in the modelling and the approximated radius of each row. In this equation the unknown parameters are: the radius of the lower row R_1, the spacing between the lower two rows c, the spacing d between the rest of the rows and the differences δx, δy of the circles centers coordinates from their initial approximated values (Fig. 4).

Table 1: Used slabs in the modelling and the approximated radius of each row

Row range	Slabs	Approximated Radius
i	j	Ri
1	1,2,…11	10,91
2	1,2,…,14	11,46
3	1,2,…,19	12,15
4	1,2,…,19	12,84
5	1,2,…,17	13,53
6	1,2,…,18	14,22
7	1,2,…,15	14,91
8	1,2,…,14	15,6
9	1,2,…,17	16,29
10	1,2,…,13	16,98
11	1,2,…,15	17,67
12	1,2,…,11	18,36

On the basis of equation (3) a system of 183 equations with 5 unknown parameters was formed. The system was solved using standard Least Square Method and a commercial mathematics software (Mathematica 5.0). The results including their standard errors and the approximations of the graphic method, which were used as input for the system of equations, are shown in the Table 2.

Table 2: Results and standard errors of the adjusted parameters and the approximations of the graphic technique

Adjusted parameters	Results of analytical technique (m)	Stnadard Error (cm)	Approximations of grafic technique
Circle Center Coordinate Difference δx	0,00358	2,44	
Circle Center Coordinate Difference δy	0,00437	2,22	
Lower Row Radius R1	10,9099	2,97	10,91
Lower two Rows Spacing c	0,548	0,91	0,55
Rest Rows Spacing d	0,692	0,08	0,686

Obviously, the parameters computed on the basis of the analytical technique are very similar to those of the graphical technique. The high accuracy of the results (standard error less than 3 cm) is due to the high redundancy of the system (183 equations- 5 unknown parameters).

4. MODELLING OF THE LINEAR WING

The three rows of slabs composing the linear wing were analytical approximated by straight lines, and their mean direction (shown by a dashed line in Fig. 4) was computed. Subsequently the direction of the chord AB and the angle formed between this chord and the mean line direction of the linear wing were calculated. This last angle was computed as 88.8^o (Fig. 4).

5. ESTIMATION OF GEOMETRIC CHARACTERISTICS OF THE THEATRE

The next step of the analysis was to calculate the arc angle of the lower row assumed to correspond to the design arc circle center for the construction of the theatre. The estimate was based on the adjusted circle center coordinates, and the coordinates of the edges of two outer slabs of the lower row (A, B in Fig. 4). These three points form a $89.6^o \pm 0.2^o$ arc. Within the errors of our data and analysis both these angles correspond to 90^o. Interestingly, the second row of seats corresponds to 35 Doric feet a measure of length in antiquity (1 Doric foot = 0.327 m; Dinsmoor, 1975).

6. CONCLUSIONS

The conclusions of our study is that the Makyneia theatre is not just a “unique” ancient theatre due to its atypical shape (Frederiksen, 2000). Its construction plan was based on a 90^o circle arc, and the corresponding chord is perpendicular to the direction of the linear wing. The design of the theatre also was made on the basis of an integer multiply of a measure of length used in the antiquity (35 Doric feet). Our results also indicate that the Makyneia theatre, as well as other atypical ancient theatres were constructed on the basis of a strict geometric plan (Gebhart, 2001) which can best be understood using field survey and analytical technique.

REFERENCES

Dinsmoor, W.B., 1975. “The architecture of ancient Greece”, B.T. Batchford, London,pp 424

Frederiksen , R., 2000.”Typology of the Greek Theatre Building in Late Classical and Hellenistic Times”, Proc. of the Danish Institute at Athens, Vol. 3, pp.135-175

Gebhard, E., 2001. “Virtrurius and the planning of the Greek theatre. In: Kallisteuma, Studies in honour of Olga Tzachou-Alexandri”, Ministry of Culture-Grrek Section of ICOM, Athens, pp.385-394

Kolona, C., 1998. “Ancient Makyneia, topopgraphy and geography” (in Greek). Unpublished Diploma Thesis, Geodesy Lab Dept. of Civil Engineering, Patras University

Kolonas, L.,1992/93. “Makyneia through the recent archaeological research” (in Greek). Naupaktiaka, 6, pp.79-95

Palyvou, C, 2001. “Notes of the geometry of the ancient theatre of Thoricos”. Achaelogisches Anzeiger, Heft 1, pp.45-48

BIOGRAPHICAL NOTES

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