Determination of Local Geoid with GPS in Trabson, Turkey

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Key words: GPS, Local Geoid, Leveling, Gravity

SUMMARY

Geoid has a closed shape that coincides with the sea level which is free of tides, currents, and similar physical forces. Modeling the geoid is realized using the geoidal undulation. The distance that lies through the ellipsoidal normal between the geoidal surface and ellipsoidal surface is called as geoidal undulation. In a network established to determine local geoid, orthometric heights are given to the points by leveling. While determining the orthometric heights, making gravimetric reductions is an important factor to find the desired geoidal undulations. The ellipsoidal height, which is necessary for finding geoidal undulation, is derived by GPS. This method called as GPS/Leveling and it is one of the most popular methods used in local geoid determination. The aim of this study is to determine the local geoid of Trabzon. A network with 39 points has been established in this region that covers an area of 30 km^2. The orthometric heights have been given to the points by leveling. Total length of the leveling routes is approximately 108 km. The gravimetric reductions have been applied to the leveling measurements and the orthometric heights with taking two points as references in Trabzon harbor. The positions and ellipsoidal heights of the points have been derived by GPS measurements. The observations have been realized by static GPS technique using dual frequency receivers and every station has been occupied at least 45 minutes. After processing the GPS observations, the precision of the position has been obtained at the level of ± 5.8 mm horizontally, and ± 7.5 mm vertically. The precision of the orthometric heights has been determined at the level of ± 5.03 mm. As shown from these results, the geoidal undulations have been determined at sub-centimeter level. As a consequence, determination of the orthometric heights of the points at sub-cm level without leveling has been tried to achieve using only GPS observations with this study.

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1. INTRODUCTION

The frequently used description of geoid surface includes idealized oceans. It is meant with this description that the oceans free of tides, currents, friction, and such physical forces, but not free of gravity (Sickle 1996).

Geoid, the surface is accepted as basic shape of the earth, is not defined sufficiently everywhere. Since it is not an ordinary geometrical surface, evaluation of geodetic measurements gets difficult. For this reason, a surface is defined that is simple in mathematical point of view, not causing difficulty in solving geodetic problems, and has little difference from geoid (Leick 1994). An ellipsoid of revolution that is flattened at the poles and has vertical axis parallel to the rotation axis of earth is used as reference surface. Deflection of the vertical ε is the angle between the normal to the ellipsoid and plumbline. Every country uses different reference ellipsoids so that the geoidal surface and ellipsoidal surface coincide sufficiently.

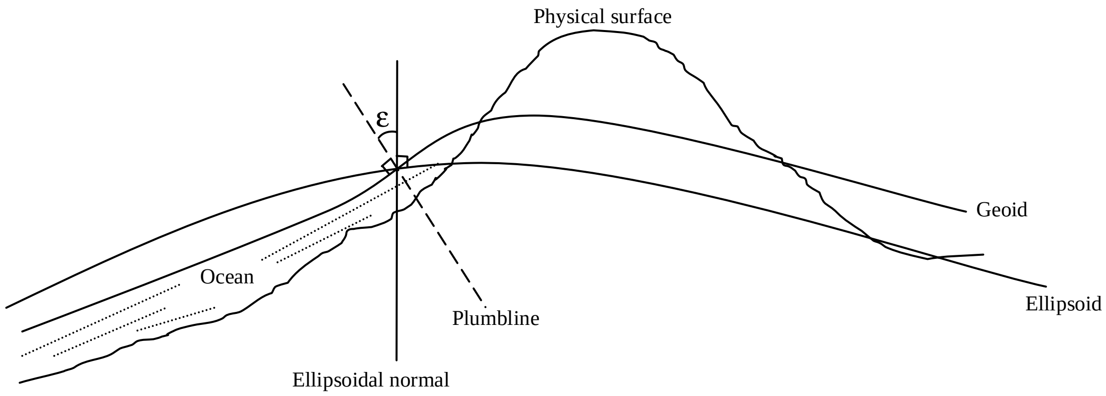

Figure 1. Geoidal, ellipsoidal, and physical surfaces

The orthometric height of a point is the distance between the geoidal surface and the plumbline that passes through that point. The relation between the orthometric height H and the ellipsoidal height h is as follows:

$$H = h - N \qquad (1)$$

where N is the geoidal undulation.

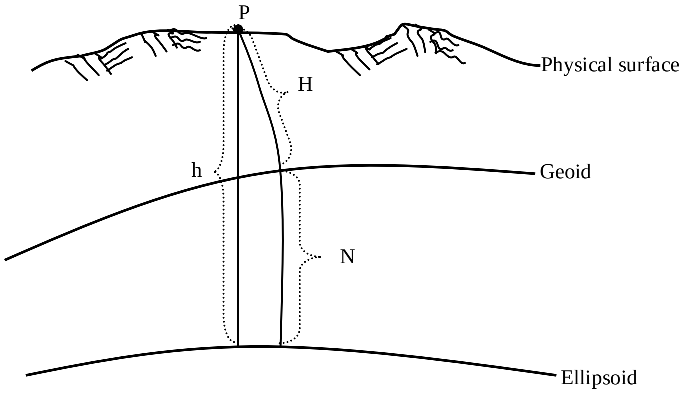

Figure 2. Geoidal, ellipsoidal heighs, and geoidal undulation

It is clear that in order to determine the geoidal undulation at a point, it is enough to know the geoidal and ellipsoidal heights of that point.

2. GPS DATA AND DATUM TRANSFORMATION

GPS has gained wide use thanks to its rapid development in geodetic control points surveying instead of terrestrial surveying techniques. GPS measurements and calculation are made in WGS 84 system and also the reference ellipsoid is WGS 84 ellipsoid. On the other hand, the European 1950 (ED 50) datum is used in Turkey and it is computed on the International ellipsoid (Hayford ellipsoid). The parameters of WGS 84 and International ellipsoids are presented in Table 1 below.

Table 1. Parameters of International and WGS 84 ellipsoid

Ellipsoid	International	WGS 84
a	6378388	6378137
b	6356911.94613	6356752.314245
1/f	1/297	1/298.257223563

In order to provide integrity of coordinate, transformation must be made between WGS 84 (X, Y, Z) and ED 50 (x, y, z) systems. This transformation can be realized by seven parameter similarity transformation: 3 translation (x_o, y_o, z_o), 3 rotation (α, β, γ), and 1 scale factor (λ) (Hofmann-Wellenhof et al. 1992).

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} + \lambda D \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad (2)$$

$$D = \begin{bmatrix} \cos\beta\cos\gamma & \cos\beta\sin\gamma + \cos\gamma\sin\beta\sin\alpha & \sin\alpha\sin\gamma + \cos\gamma\sin\beta\cos\alpha \\ -\sin\gamma\cos\beta & \cos\alpha\cos\gamma - \sin\alpha\sin\gamma\sin\beta & \sin\alpha\cos\gamma + \cos\alpha\sin\gamma\sin\beta \\ \sin\beta & -\sin\alpha\cos\beta & \cos\beta\cos\alpha \end{bmatrix} \quad (3)$$

The transformation parameters are calculated from at least 3 common points whose coordinates are known in both coordinate systems. When the equation (2) is applied to the weight center of the common points the following equations are obtained.

$$\begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} + \lambda D \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} - \lambda D \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \quad (5)$$

Here, (X_s, Y_s, Z_s) and (x_s, y_s, z_s) are the coordinates of weight center in each system. The shifted coordinates of points are calculated as follows:

$$\begin{aligned} \overline{X}_i &= X_i - X_s \;, \quad \overline{Y}_i = Y_i - Y_s \;, \quad \overline{Z}_i = Z_i - Z_s \\ \overline{x}_i &= x_i - x_s \;, \quad \overline{y}_i = y_i - y_s \;, \quad \overline{z}_i = z_i - z_s \end{aligned} \quad (6)$$

Now the equation (2) changes in the following form:

$$\begin{bmatrix} \overline{x}_i \\ \overline{y}_i \\ \overline{z}_i \end{bmatrix} = \lambda D \begin{bmatrix} \overline{X}_i \\ \overline{Y}_i \\ \overline{Z}_i \end{bmatrix} \quad (7)$$

The solution of normal equations yields α, β, γ, and λ. The coordinate transformation is realized substituting these values in equation (2). Then, the transformation from Cartesian coordinates to Geographic coordinates is performed using following equations.

$$B = \arctan\left[\frac{z}{\sqrt{x^2 + y^2}} \left(1 - \frac{e^2 N}{N + h}\right)^{-1} \right] \quad (8)$$

$$L = \arctan\left(\frac{y}{x} \right) \quad (9)$$

$$h = \frac{\sqrt{x^2 + y^2}}{\cos\varphi} - N \quad (10)$$

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 B}} \quad (11)$$

Here, N is radius of curvature in direction perpendicular to the prime vertical, B is geographical latitude, and L is geographical longitude. B and h are calculated iteratively (Leick 1994). The first value of B is obtained as follows:

$$B_o = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}\left(1 - e^2\right)}\right) \quad (12)$$

3. DEFINITION OF GRAVITY

The sum of mass attraction to a constant object on earth and the centrifugal force is named gravity. Gravity is a scalar value. The direction of gravity vector lies into plumbline. So the plumbline is affected by field structure. In order to find the orthometric height of a point, the gravity value at that point is necessary.

In gravity measurement, gravimeter is used and the observations of it are as dial readings. In order to get the dial readings in miligal, they must be multiplied by calibration constant that is in miligal. It is necessary to check the calibration constant occasionally. The verification of this constant is provided by making measurements at two reference points whose gravity values are known (Torge 1980). The calibration constant equals to the ratio between the difference of the gravity values and the difference of the dial readings.
The gravity value g_Q of a point Q that lies in the same plumbline with point P on the physical surface of the Earth is obtained by Poincaré and Prey reduction.

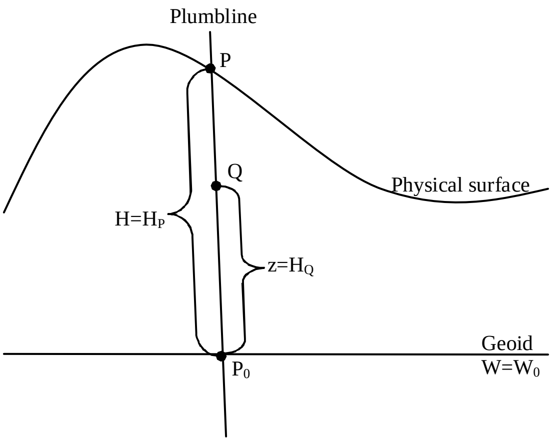

Figure 3. Prey reduction

$$g_Q = g_P - \int_Q^P \frac{\partial g}{\partial h} dH \quad (13)$$

$$\frac{\partial g}{\partial h} = -2gJ + 4\pi k\rho - 2\omega^2 \quad (14)$$

$$\frac{\partial \gamma}{\partial h} = -2\gamma J_0 - 2\omega^2 \quad (15)$$

As $\rho = 2.67$ g/cm^3 and $k = 66.7*10^{-9}$ c.g.s., the following equation is written:

$$\frac{\partial g}{\partial h} = -0.3086 + 0.2238 = -0.0848 \text{ gal/km} \quad (16)$$

In this case, the equation (13) can be written as follows:

$$g_Q = g_P + 0.0848\ (H_P - H_Q) \quad (17)$$

where the unit of g is gal and unit of H is km (Heiskanen and Moritz 1966).

4. GEOMETRIC LEVELING AND ORTHOMETRIC HEIGHT COMPUTATION

Geometric leveling is based on the procedure that taking the differences of rod readings at two points that remote from each other (Fig. 4).

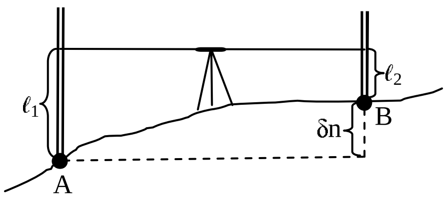

Figure 4. Leveling

$$\delta n = \ell_1 - \ell_2 \quad (18)$$

The sum of differences of rod readings between two points yields the difference in elevation (Fig. 5).

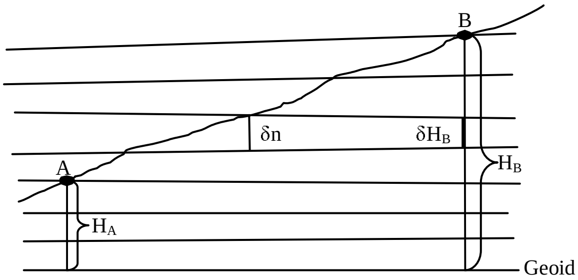

Figure 5. Leveling and orthometric height

$$\Delta n_{AB} = \sum_{A}^{B} \delta n = \int_{A}^{B} dn \qquad (19)$$

Even though there is no measurement error in a closed leveling route, it is seen that the algebraic sum of the elevation differences does not exactly equal to zero. It can be seen that leveling is not straightforward as explained before; indeed it is a sophisticated surveying procedure. Since leveling surfaces are not parallel to each other, δn differences obtained from geometric leveling are different from δH_B. Therefore, the sum of the elevation differences between the point A and point B does not equal to the orthometric differences of those points. If the increment in potential W is shown δW, the following equation can be written

$$\delta W = -g\delta n \qquad (20)$$

where, g is the gravity at instrument point. There is no direct geometric relation between orthometric height and the leveling result. If the gravity is measured along with leveling and considering equation 20, following can be written

$$W_B - W_A = -\sum_{A}^{B} g\delta n \qquad (21)$$

Hence potential difference, which is physical quantity, is computed (Heiskanen and Moritz 1966). The potential difference between the point O on the geoid and the point A on the physical surface is

$$C = W_0 - W_A = \int_{0}^{A} g dn \qquad (22)$$

where, C is geopotential number of the point A.

C is free from leveling route. The unit of the C is g.p.u.

$$1 \text{ g.p.u.} = 1 \text{ kgal metre} = 1000 \text{ gal metre} \qquad (23)$$

The dynamic heights are commonly used instead of geopotential number as

$$H^{din} = C/\gamma_o \tag{24}$$

where, γ_o is usually the normal gravity at 45^o latitude. The value of the $\gamma_{45}{}^o$ equals to 980.6294 mgal.

Sometimes, it is convenient to transform Δn_{AB} to dynamic height difference by adding small corrections as follows

$$\begin{aligned}\Delta H_{AB}^{din} = H_B^{din} - H_A^{din} &= \frac{1}{\gamma_0}(C_B - C_A) = \frac{1}{\gamma_0}\int_A^B g dn = \frac{1}{\gamma_0}\int_A^B (g - \gamma_0 + \gamma_0) dn \\ &= \underbrace{\int_A^B dn}_{\Delta n_{AB}} + \underbrace{\int_A^B \frac{g-\gamma_0}{\gamma_0} dn}_{DC_{AB}}\end{aligned} \tag{25}$$

The equation (24) can be written as follows:

$$\Delta H_{AB}^{din} = \Delta n_{AB} + DC_{AB} \tag{26}$$

Here DC_{AB} is the dynamic correction (Heiskanen and Moritz 1966)

$$DC_{AB} = \int_A^B \frac{g-\gamma_0}{\gamma_0} dn = \sum_A^B \frac{g-\gamma_0}{\gamma_0} \delta n \tag{27}$$

Let the intersection point of the plumbline which also passes through the point P with the geoid be P_0 (Fig. 3). The potential number of P equals the difference of the potential W at P and the potential W_0 at P_0.

$$C = W_0 - W \tag{28}$$

Taking into consideration the equation (22), the potential number C, which is free from the leveling route, can be written as follows:

$$C = \int_0^H g dH \tag{29}$$

The equation (29) includes H in a closed form. The open form of H

$$dC = -dW = g dH \tag{30}$$

$$dH = -\frac{dW}{g} = \frac{dC}{g} \tag{31}$$

$$H = -\int_{W_0}^{W} \frac{dW}{g} = \int_{0}^{C} \frac{dC}{g} \tag{32}$$

H can be obtained by rearranging the equation (29) slightly.

$$C = \int_{0}^{H} g dH = H \frac{1}{H} \int_{0}^{H} g dH = \bar{g} H \tag{33}$$

Here, $\bar{g}$ is the mean value of gravity throughout the point P_0 and point P.

$$\bar{g} = \frac{1}{H} \int_{0}^{H} g dH \tag{34}$$

$$H = \frac{C}{\bar{g}} \tag{35}$$

$$\bar{g} = \frac{1}{H} \int_{0}^{H} g(z) dz \tag{36}$$

Here g(z) is the true gravity at point Q with height z. From Prey reduction

$$g(z) = g + 0.0848\ (H-z) \tag{37}$$

Substituting the equation (37) in the equation (36)

$$\bar{g} = \frac{1}{H} \int_{0}^{H} [g + 0.0848(H - z)] dz = \frac{1}{H} gz + \frac{1}{H} 0.0848 \left[Hz - \frac{z^2}{2} \right]_0^H \tag{38}$$

$\bar{g}$ can be expressed shortly such as

$$\bar{g} = g + 0.0424 * H \tag{39}$$

The following equation can be written using g_0 gravity obtained from Prey reduction.

$$\bar{g} = \frac{1}{2}(g + g_0) \tag{40}$$

This usage was proposed by Mader (1954) and it is assumed that $\bar{g}$ changes linearly along plumbline. Substituting the equation (40) in equation (35), the orthometric height of point P is determined.

$$H = \frac{C}{g + 0.0424H} \quad (C : g.p.u.\ ,\ g : gal\ ,\ H : km) \tag{41}$$

When height is being transferred from one point to another, orthometric correction must be applied to the height difference. In order to find this correction, following approach can be used.

$$\Delta H_{AB} = H_B - H_A \tag{42}$$

If the dynamic heights for points A and B are added to and subtracted from equation (42), this equation takes the following form.

$$\Delta H_{AB} = H_B - H_A + H_A^{din} - H_A^{din} + H_B^{din} - H_B^{din} = H_B^{din} - H_A^{din} + H_A^{din} - H_A - H_B^{din} + H_B \tag{43}$$

$$\Delta H_{AB} = \Delta H_{AB}^{din} + (H_A^{din} - H_A) - (H_B^{din} - H_B) \tag{44}$$

Substituting the equation (26) in the equation (44) yields the following equations.

$$\Delta H_{AB}^{din} = \Delta n_{AB} + DC_{AB} \tag{45}$$

$$\Delta H_{AB} = \Delta n_{AB} + DC_{AB} + DC_{A_0A} - DC_{B_0B} \tag{46}$$

$$\Delta H_{AB} = \Delta n_{AB} + OC_{AB} \tag{47}$$

Here, OC_{AB} is orthometric correction. The expanded form of the equation (47) is

$$\Delta H_{AB} = \Delta n_{AB} + \sum_A^B \frac{g - \gamma_0}{\gamma_0}\delta n + \frac{\bar{g}_A - \gamma_0}{\gamma_0} H_A - \frac{\bar{g}_B - \gamma_0}{\gamma_0} H_B \tag{48}$$

Height of point B that includes the orthometric correction is

$$H_B = H_A + \Delta H_{AB} \tag{49}$$

5. APPLICATION

Trabzon municipality boundary is selected as the studying area. In order to determine local geoid at cm level, a leveling network has been established. Considering the need of gravity values and homogeneity of the points in the network, distances between points are limited about 1 km. The leveling network has 41 points. The precise two-way leveling measurements have been made with Topcon 101C digital level. The benchmarks DN2 and DN3 are taken reference points in leveling network. The leveling network has 20 loops and total length of the leveling routes is about 108 km. The maximum computed loop closure is 9.6 mm for the loop length of 12.759 km. The a priori standard deviation s_0 is 3.206 mm. The a posteriori standard deviation obtained from least squares adjustment is 2.353 mm. The test statistic T is 1.856 and the critical value q is 2.465 of the global test. Since $T<q$, the global test is passed. After the global test, Tau test has been performed. The critical value of this test is 1.936 and the test statistics of the measurements are between 0.203~1.901. So there is no outlying measurement in the leveling network.

The gravity measurements of points have been made by Worden Gravimeter (No 801, Model III). The points BG-4087 and BG-4088 at Trabzon Harbor were the reference points. After determination of gravity values of the points, the mean gravity values have been calculated by means of Prey reduction.

The heights obtained from the first adjustment have taken as approximate values and they have been recomputed by applying geophysical reductions (Table 2) to them. The a posteriori standard deviation m_0 after final adjustment is ±5.03 mm and the standard deviation values m_i for observations changes between ±4.593 mm ~ ±12.279.

The horizontal positions and ellipsoidal heights have been determined by GPS measurements. The observations have been realized with 2 Ashtech Z-Xtreme and 3 Ashtech Z-Surveyor dual-frequency GPS receivers. The total session number is 28 and the measurements were completed in 7 days. The occupation time at each point was at least 45 minutes. The observations have been processed with taking G_09, G_11, G_12, G_28, and TRAB GPS permanent station as fixed points in GeoGenius2000 program (Fig. 6). The precisions are 2.5 mm at horizontal, 3.5 mm at vertical in baseline processing, and 5.8 mm at horizontal, 7.5 mm at vertical in network adjustment.

Table 2. Dynamic corrections, orthometric corrections, and corrected height differences

First Point	Last point	Δn_{AB} (m)	DC_{AB} (m)	DC_A (m)	DC_B (m)	OC_{AB} (m)	ΔH_{AB} (m)	First Point	Last Point	Δn_{AB} (m)	DC_{AB} (m)	DC_A (m)	DC_B (m)	OC_{AB} (m)	ΔH_{AB} (m)
G_02	G_01	1,5647	-0,0006	-0,0021	-0,0026	0,0000	1,5647	G_33	G_34	-40,4427	0,0175	-0,1064	-0,0875	-0,0014	-40,4441
G_03	G_02	0,7486	-0,0003	-0,0019	-0,0021	0,0000	0,7485	G_34	G_35	-6,3901	0,0027	-0,0875	-0,0842	-0,0006	-6,3908
G_04	G_03	-3,8026	0,0014	-0,0032	-0,0019	0,0000	-3,8025	G_35	G_36	76,6379	-0,0333	-0,0842	-0,1208	0,0034	76,6412
G_05	G_04	4,8492	-0,0017	-0,0015	-0,0032	0,0000	4,8492	G_37	G_38	-48,6305	0,0238	-0,1362	-0,1139	0,0015	-48,6290
G_05	G_06	0,9959	-0,0004	-0,0015	-0,0018	0,0000	0,9959	G_01	G_26	174,1287	-0,0676	-0,0027	-0,0752	0,0050	174,1337
G_07	G_06	-0,2717	0,0001	-0,0019	-0,0018	0,0000	-0,2717	G_02	G_25	200,7108	-0,0790	-0,0021	-0,0877	0,0066	200,7173
G_07	G_08	-0,4110	0,0001	-0,0019	-0,0018	0,0000	-0,4110	G_03	G_24	122,5895	-0,0469	-0,0019	-0,0511	0,0023	122,5917
G_08	G_09	3,8559	-0,0014	-0,0018	-0,0032	0,0000	3,8559	G_04	G_23	130,1725	-0,0490	-0,0032	-0,0542	0,0020	130,1745
G_09	G_10	3,1312	-0,0012	-0,0032	-0,0044	0,0000	3,1312	G_22	G_05	-110,9186	0,0422	-0,0459	-0,0015	-0,0022	-110,9208
G_10	G_11	21,5718	-0,0081	-0,0044	-0,0127	0,0002	21,5719	G_21	G_07	-81,0748	0,0300	-0,0328	-0,0019	-0,0009	-81,0756
G_11	G_12	11,3715	-0,0044	-0,0127	-0,0174	0,0003	11,3718	G_09	G_19	122,8260	-0,0473	-0,0032	-0,0524	0,0019	122,8279
G_12	G_13	-2,5535	0,0010	-0,0174	-0,0165	0,0001	-2,5534	G_10	G_18	95,9929	-0,0367	-0,0044	-0,0421	0,0009	95,9938
G_13	G_14	-3,9632	0,0015	-0,0165	-0,0150	0,0000	-3,9631	G_12	G_17	145,3098	-0,0586	-0,0174	-0,0779	0,0019	145,3117
G_14	G_15	69,2152	-0,0281	-0,0150	-0,0449	0,0018	69,2170	G_13	G_16	115,7296	-0,0480	-0,0165	-0,0683	0,0038	115,7334
G_16	G_15	-50,4776	0,0217	-0,0683	-0,0449	-0,0017	-50,4792	G_15	G_38	129,5548	-0,0591	-0,0449	-0,1139	0,0098	129,5646
G_17	G_16	-32,1337	0,0138	-0,0779	-0,0683	0,0042	-32,1295	G_16	G_37	127,7078	-0,0592	-0,0683	-0,1362	0,0086	127,7164
G_18	G_17	82,2602	-0,0334	-0,0421	-0,0779	0,0024	82,2626	G_17	G_36	89,2741	-0,0385	-0,0779	-0,1208	0,0044	89,2785
G_20	G_19	-120,0467	0,0504	-0,1068	-0,0524	-0,0040	-120,0507	G_18	G_35	94,8965	-0,0388	-0,0421	-0,0842	0,0033	94,8997
G_21	G_20	165,2428	-0,0676	-0,0328	-0,1068	0,0064	165,2491	G_19	G_34	77,5847	-0,0322	-0,0524	-0,0875	0,0029	77,5875
G_22	G_23	24,1031	-0,0096	-0,0459	-0,0542	-0,0013	24,1018	G_20	G_33	-2,0194	0,0009	-0,1068	-0,1064	0,0005	-2,0189
G_23	G_24	-11,3856	0,0046	-0,0542	-0,0511	0,0014	-11,3842	G_21	G_32	14,6051	-0,0057	-0,0328	-0,0394	0,0009	14,6059
G_24	G_25	78,8699	-0,0331	-0,0511	-0,0877	0,0035	78,8734	G_22	G_31	-2,0536	0,0008	-0,0459	-0,0438	-0,0013	-2,0548
G_25	G_26	-25,0174	0,0107	-0,0877	-0,0752	-0,0018	-25,0191	G_23	G_30	123,0442	-0,0527	-0,0542	-0,1178	0,0109	123,0550
G_26	G_27	180,4345	-0,0829	-0,0752	-0,1742	0,0160	180,4505	G_24	G_29	67,0972	-0,0282	-0,0511	-0,0833	0,0040	67,1012
G_28	G_27	2,2016	-0,0011	-0,1736	-0,1742	-0,0005	2,2010	G_29	G_40	199,1671	-0,0946	-0,0834	-0,1956	0,0177	199,1847
G_29	G_28	164,9883	-0,0771	-0,0834	-0,1736	0,0132	165,0014	G_30	G_40	131,8345	-0,0642	-0,1178	-0,1956	0,0137	131,8482
G_30	G_29	-67,3326	0,0302	-0,1178	-0,0833	-0,0042	-67,3368	G_08	DN2	28,0248	-0,0102	-0,0018	-0,0121	0,0001	28,0249
G_31	G_30	149,2008	-0,0636	-0,0438	-0,1178	0,0104	149,2111	DN3	G_21	54,2384	-0,0203	-0,0117	-0,0328	0,0008	54,2391
G_31	G_32	-11,9177	0,0047	-0,0438	-0,0394	0,0003	-11,9174	G_20	DN2	-218,7038	0,0880	-0,1068	-0,0121	-0,0067	-218,7105
G_32	G_33	148,6184	-0,0618	-0,0394	-0,1064	0,0052	148,6236	DN3	G_22	82,8147	-0,0318	-0,0117	-0,0458	0,0023	82,8169

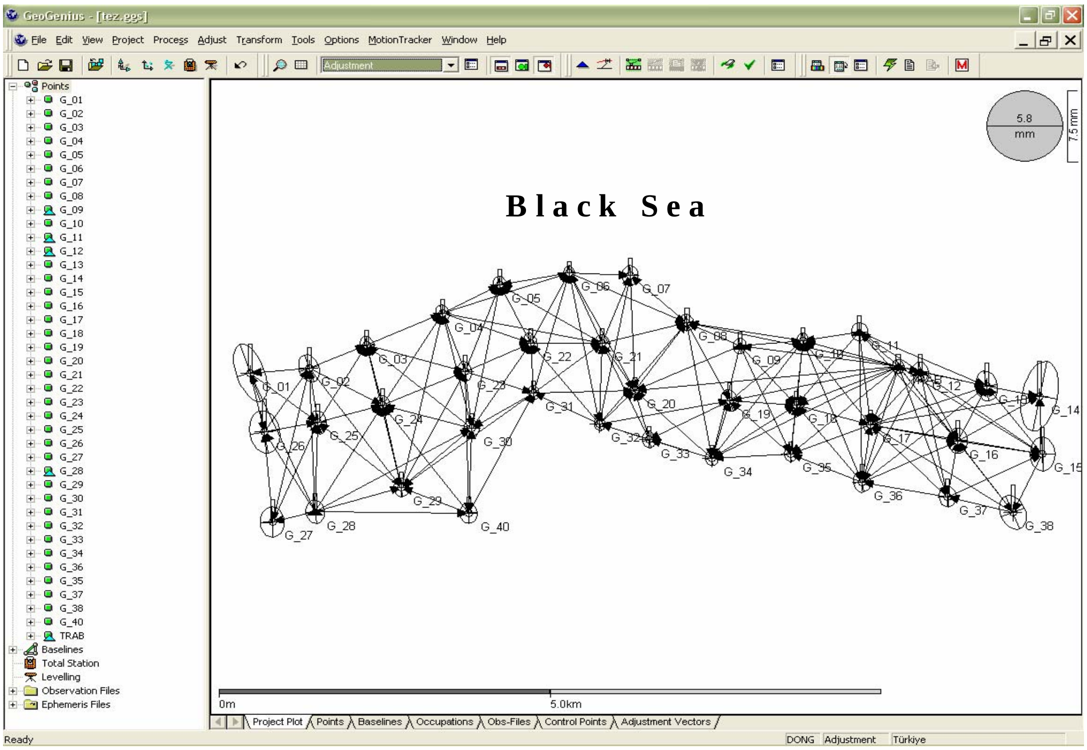

Figure 6. The GPS network

The ellipsoidal heights and coordinates from GPS measurements, the orthometric heights from leveling, and the geoidal undulations from equation (1) are presented in Table3.

Table 3. The coordinates, ellipsoidal, and orthometric heights of the points

Point	Y	X	h	H	N	Point	Y	X	h	H	N
G_01	555488.856	4540356.676	-2.872	7.483	-10.355	G_21	560806.330	4540845.695	75.726	86.461	-10.735
G_02	556373.551	4540433.848	-4.519	5.916	-10.435	G_22	559717.856	4540844.823	104.338	115.036	-10.698
G_03	557246.508	4540809.285	-5.376	5.171	-10.547	G_23	558732.021	4540384.431	128.624	139.141	-10.517
G_04	558384.794	4541334.534	-1.690	8.966	-10.656	G_24	557489.760	4539813.543	117.400	127.758	-10.358
G_05	559260.344	4541806.963	-6.661	4.121	-10.782	G_25	556509.390	4539535.249	196.349	206.626	-10.277
G_06	560304.723	4542012.040	-5.783	5.117	-10.900	G_26	555705.779	4539378.892	171.425	181.611	-10.186
G_07	561221.688	4541965.170	-5.550	5.381	-10.931	G_27	555822.951	4537891.165	352.099	362.048	-9.949
G_08	562087.623	4541190.213	-5.632	4.974	-10.606	G_28	556467.470	4538058.420	349.818	359.842	-10.024
G_09	562882.460	4540800.130	-1.224	8.832	-10.056	G_29	557770.451	4538461.966	184.719	194.853	-10.134
G_10	563843.164	4540884.632	1.289	11.964	-10.675	G_30	558831.058	4539436.436	251.838	262.185	-10.347
G_11	564690.010	4541032.150	23.404	33.537	-10.133	G_31	559763.805	4540031.518	102.485	112.982	-10.497
G_12	565607.010	4540372.180	34.783	44.904	-10.121	G_32	560760.649	4539510.760	91.341	101.066	-9.725
G_13	566607.513	4540139.183	31.677	42.361	-10.684	G_33	561512.506	4539250.875	240.002	249.683	-9.681
G_14	567406.267	4539972.520	28.248	38.395	-10.147	G_34	562461.545	4538943.541	198.988	209.239	-10.251
G_15	567453.864	4539011.440	97.123	107.607	-10.484	G_35	563647.312	4539012.677	192.543	202.858	-10.315
G_16	566174.318	4539234.043	147.622	158.084	-10.462	G_36	564726.894	4538548.795	269.219	279.494	-10.275
G_17	564852.113	4539502.247	179.762	190.223	-10.461	G_37	566022.413	4538310.553	275.468	285.788	-10.320
G_18	563739.829	4539824.566	97.512	107.959	-10.447	G_38	567007.737	4538046.494	226.829	237.166	-10.337
G_19	562735.211	4539897.517	121.205	131.657	-10.452	G_40	558783.723	4538038.808	383.939	394.023	-10.084
G_20	561291.425	4540066.786	241.311	251.701	-10.390						

The geoidal surface of the working region has been formed using the coordinates and geoidal undulations of the points in ArcView 3.2 program (Fig. 7).

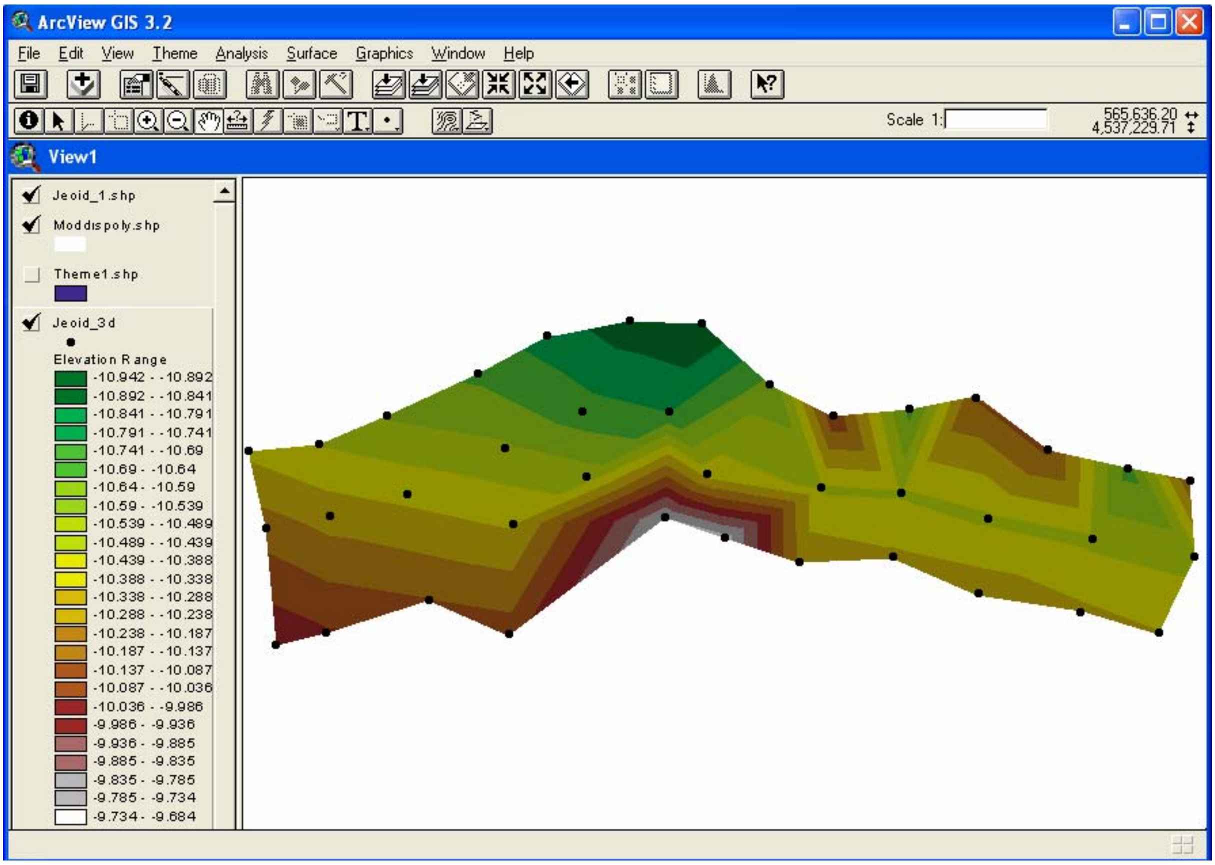

Figure 7. Local geoidal surface in Trabzon/Turkey

6. CONCLUSIONS

In this work, the precision of the ellipsoidal heights has been obtained at ±7.5 mm from GPS network adjustment and the precision of orthometric heights has been obtained at ±5.03 mm from leveling adjustment. Using obtained values above, the geoidal undulations N_i have been calculated between -10.355 m and -9.681 m with precision of ±9.03 mm. Consequently, it is seen that the orthometric height of any point in the working area can be determined with a precision of centimeter or sub-centimeter level using only GPS measurements and the determined geoid model. Since there is no need to make leveling measurements to determine the orthometric heights, many savings will be got in time, effort, and cost point of view. Additionally, the other earth sciences such as geology and geophysics may also use the determined geoid in their studies.

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