Progress Towards a cm Geoid for Dubai Emirate

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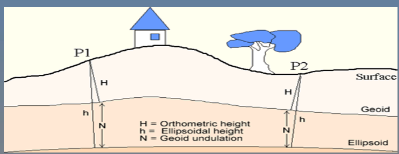
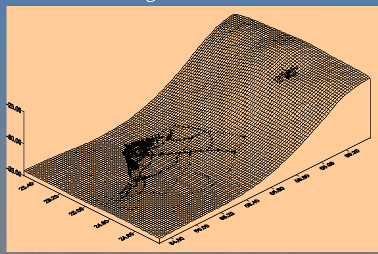


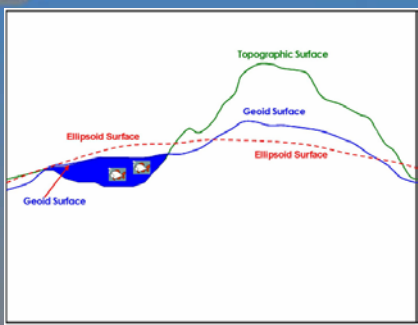
Definition of the Geoid

Geoid is the equipotential surface of the gravity field of the earth.

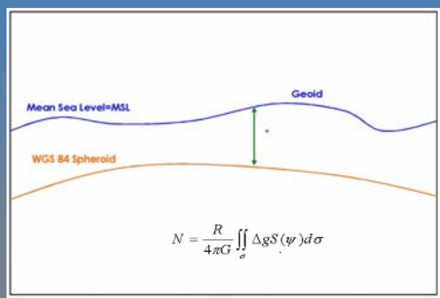
Geoid : Level surface of global undisturbed oceans



Earth Surfaces

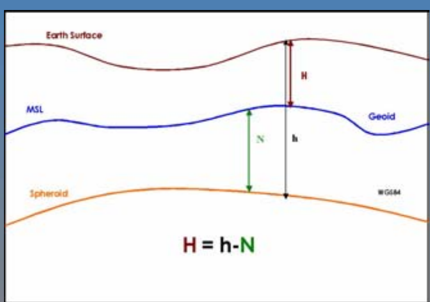


Geoid – Ellipsoid Separation



$$N = \frac{R}{4\pi G}\iint_{\sigma} \Delta g S(\psi) d\sigma$$

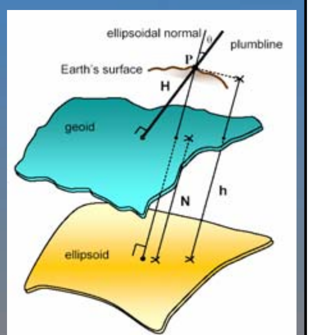
GPS Heighting



Spirit levelling Vs GPS

- Traditional means for establishing vertical control (H): spirit-levelling
  - costly
  - labourious
  - inefficient, difficult in remote areas, mountainous terrain, over large regions
- With advent of satellite-based global positioning systems (GPS) 3D positioning has been revolutionized

h – H – N = 0

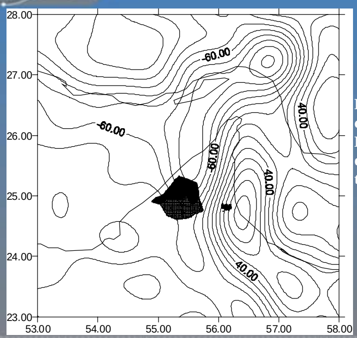


Why to combine h, H and N ??

- modernize regional vertical datums
- unify/connect national regional datums between neighbouring countries
- transform between different types of height data (GPS-levelling)
- refine and test existing gravimetric geoid models
- better understanding of data error sources
  - calibrate geoid error model
  - assess noise in GPS heights, test a-priori error measures
  - evaluate levelling precision, test a-priori error values
- Other applications: sea level change monitoring, post-glacial rebound studies, etc.

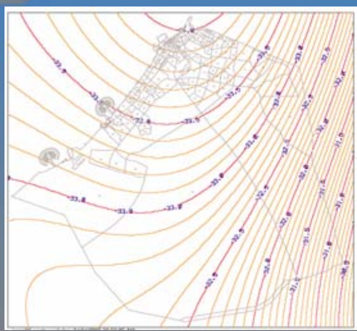
Problems of combining heterogeneous heights

- random errors in the derived heights *h*, *H*, and *N*
- datum inconsistencies inherent among the height types
- systematic effects and distortions (long-wavelength geoid errors, poorly modelled GPS errors and over-constrained levelling network adjustments)
- assumptions/theoretical approximations made in processing observed data (neglecting sea surface topography or river discharge corrections at tide gauges)
- approximate or inexact normal/orthometric height corrections
- instability of reference station monuments over time (geodynamic effects, land uplift/subsidence)

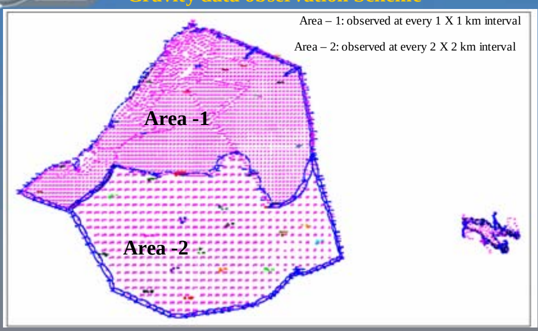


EGM96 gravity data (20 mgal contour interval).
Hatta is obviously on the edge of a major tectonic gravity trend.

EGM96 – GEOID MODEL



Gravity data observation Scheme



Statistics of Gravity Data Reduction

Statistics of gravity data reductions

All data (9206 pts)

Unit: mgal	Mean	Std.dev.	Min	Max
Original data	-54	40.1	-128	132
- EGM	-5	13	-78.8	62.3
- EGM – RTM (60")	-2.7	12.3	-70.6	66.9
- EGM – RTM (30")	-3.9	12.4	-78.4	69.9
- EGM – Isostatic	-0.6	13.5	-69.1	66.7

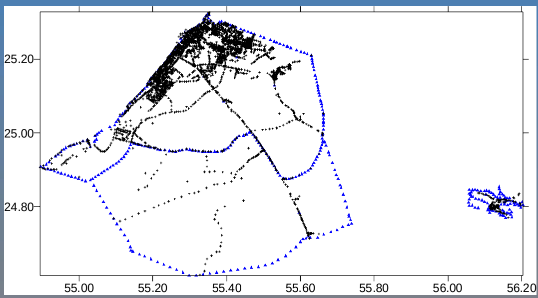
Dubai main area gravimetry (2881 pts)

Unit: mgal	Mean	Std.dev.	Min	Max
Original data	-72	10.9	-95.6	51
- EGM	-8.8	11.2	-40.5	8.3
- EGM – RTM (60")	-6.5	11.3	-44.4	10.9
- EGM – RTM (30")	-8.2	10.9	-32.6	9.1
- EGM – Isostatic	-5.6	11.2	-52.5	13.7

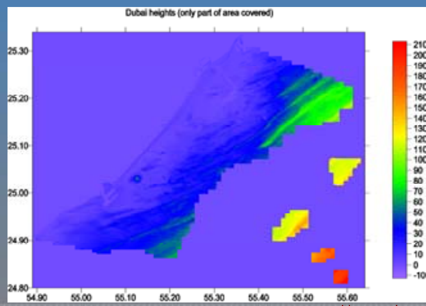
Hatta gravimetry only (199 pts)

Unit: mgal	Mean	Std.dev.	Min	Max
Original data	66.5	19	44.7	132
- EGM	-18	18.1	-42.7	62.3
- EGM – RTM (60")	-22	16.3	-45.4	32.7
- EGM – RTM (30")	-11	15.8	-33.9	40.1
- EGM – Isostatic	-31	16.1	-54.4	22.6

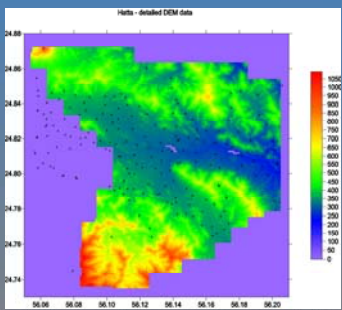
Available GPS-leveling geoid data. Black: DM data; blue triangles: Levelling data.



Area covered by detailed DEM data – mainland Dubai



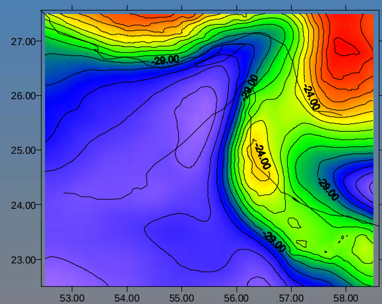
Area covered with 100 m DEM data – Hatta



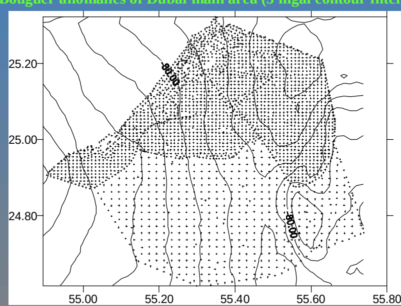
Comparisons of gravimetric geoid to Dubai Municipality GPS leveling

Unit: m	Mean	Std.dev.
Dubai area (3157 GPS pts) Gravimetric geoid	-1.40	0.09
EGM96 only	-0.83	0.16
Hatta area (110 pts) Gravimetric geoid	-1.39	0.12
EGM96	-1.52	0.25

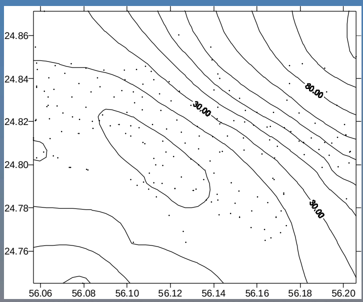
Gravimetric geoid of the Dubai region, from EGM96, gravimetry and DEM's



Bouguer anomalies of Dubai main area (5 mgal contour interval)



Bouguer anomalies of Hatta (10 mgal contour interval)



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The gravimetric geoid computation

The Dubai precise geoid has been computed in two steps

- A gravimetric geoid model, computed by spherical FFT in a global datum
- A GPS-tailored local geoid, which fits the GPS observations and the Dubai vertical datum to a few cm. This step has involved an iterative editing of GPS-leveling outliers.

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Why to use FFT ?

- FFT provides very fast evaluation of convolution sums/integrals with gridded data
- In planar approximation, the Stokes and terrain correction integrals are convolutions
- In spherical approximation, these integrals are convolutions along the parallels, and so are the summations for the GM-contributions
- Gravity and topography data are usually provided on regular grids
- Computations for very large areas can be performed on a PC

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Geoid undulations by FFT

Computational Procedure :

- Subtract effect of GM from Δg (long wavelength)
- Subtract effect of terrain from Δg (short wavelength)
- Use the reduced Δg in the FFT formulas
- Add to the results (reduced co-geoid) the GM effect
- Add to the results (reduced co-geoid) the indirect terrain effects

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Gravimetric Geoid Computation

The Geoid signal N is constructed by subdividing it into three parts

$$N = N_1 + N_2 + N_3$$

Where

N_1 = Spherical harmonic expansion complete to degree and order 360

N_2 = From the topography

N_3 = From contributions of residual gravity

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Long wave length Geoid contribution

$$\Delta g_P^{GM} = G\sum_{n=2}^{n_{max}}(n-1)\sum_{m=0}^{n}[C_{nm}\cos m\lambda_P + S_{nm}\sin m\lambda_P]P_{nm}(\sin\varphi_P)$$

$$N_P^{GM} = R\sum_{n=2}^{n_{max}}\sum_{m=0}^{n}[C_{nm}\cos m\lambda_P + S_{nm}\sin m\lambda_P]P_{nm}(\sin\varphi_P)$$

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Medium wave length Geoid contribution

$$N(x_k, y_l) = \frac{1}{2\pi\gamma} F^{-1}\{F\{\overline{\Delta g}(x_k, y_l)\} F\{\overline{l_N}(x_k, y_l)\}\} = \frac{1}{2\pi\gamma} F^{-1}\{\overline{\Delta G}(u_m, v_n)\, \overline{L_N}(u_m, v_n)\}$$

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Short wave length Geoid contribution

$$\delta N_P \approx -\frac{\pi k \rho}{\gamma} H_P^2 - \frac{k\rho}{6\gamma} \iint_E \frac{H^3 - H_P^3}{l^3} dxdy$$

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Fitting the geoid to GPS/levelling data

$\varepsilon = N_{grav} - N_{GPS} = f(\varphi,\lambda) + \varepsilon'$

The basic principle used here is to model the gravimetric and GPS geoid difference by a smooth function consisting of a trend function f (a polynomial) and a residual ε', to be modeled by least/squares collocation

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Problem Formulation

Standard practice: Use of a corrector surface to model the datum discrepancies and systematic effects when combining GPS, geoid and orthometric heights

Theory: $h_i - H_i - N_i = 0 \rightarrow N_i^{GPS/levelling} = N_i$

Practice: $h_i - H_i - N_i = l_i \rightarrow N_i^{GPS/levelling} \neq N_i$

Model: $l_i = h_i - H_i - N_i = \mathbf{a}_i^T \mathbf{x} + v_i$

residuals

parametric model

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Improving the GPS Heighting

$H_A = h_A - N_A$ or $h_A - H_A - N_A = 0$ Eqn-1

Where, H_A = orthometric height, h_A =ellipsoidal height, N_A = Geoidal Separation

$h_i - H_i - N_i = c_i^T x + v_i$ Eqn-2

$\mathbf{c}_i^T \mathbf{x} = \cos\varphi_i \cos\lambda_i\, x_1 + \cos\varphi_i \sin\lambda_i\, x_2 + \sin\varphi_i\, x_3 + x_4$ Eqn-3

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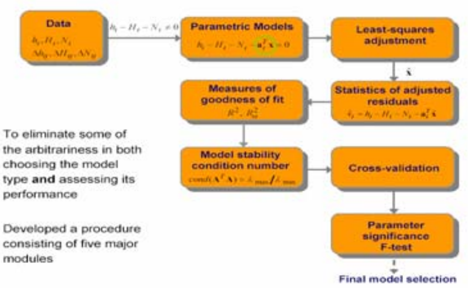
Semi-automated Parametric Model Testing Procedure



- To eliminate some of the arbitrariness in both choosing the model type **and** assessing its performance
- Developed a procedure consisting of five major modules

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R.M.S. fit of GPS leveling data to the geoid

Unit: m	Mean	Std.dev.	Min	Max
All data (3540 pts.)	-0	0.041	-0.269	0.245
DM GPS data, Dubai main (3212 pts)	0	0.036	-0.215	0.198
Hansa GPS data, Dubai (150 pts)	0	0.05	-0.115	0.21
Hatta area (110 pts.)	-0.01	0.11	-0.423	0.461

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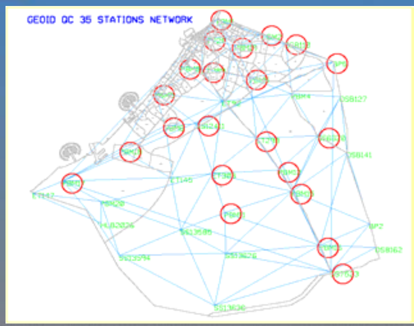
Testing the Geoid Model



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MAIN-LAND

Accuracy tested for stations from main-data base

method of observation	Accuracy winthin 5 cm	%	Accuracy more than 5 cm	%	total station
fast static, kinematic	2857	87	440	13	3297
static method (more than 1 h)	318	87	49	13	367

Accuracy tested from the network which observed to test the GM

Total Number of Stations observed	35	%
Stations with +/- 2cm	25	71
Stations with +/- 3cm	4	11
Stations with +/- 4cm	4	12
Stations with +/- 5cm more	1	3
Outliar	1	3

Note : 100% Station are within 5 cm

HATTA

method of observation	Accuracy winthin 10 cm	%	Accuracy more than 10 cm	%	total station
static, fast static, kinematic	99	86	16	14	115

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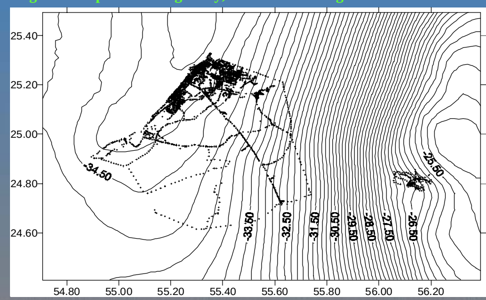
Dubai geoid computed from gravity, GPS and leveling.Contour interval 20 cm.



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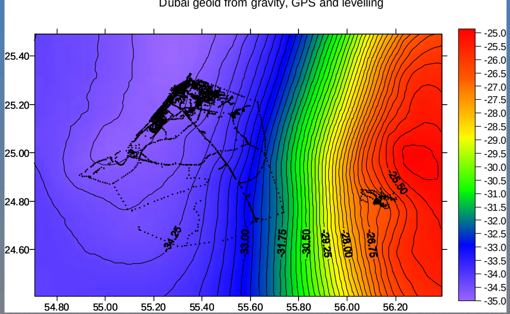
Dubai Geoid form gravity, GPS and Levelling



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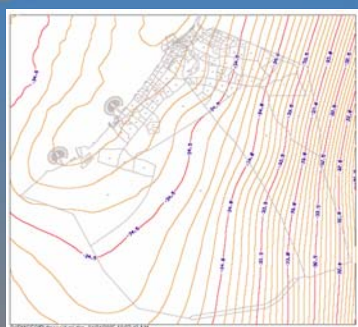
DM-GEOID MODEL



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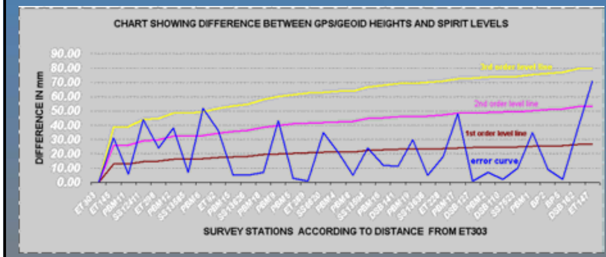

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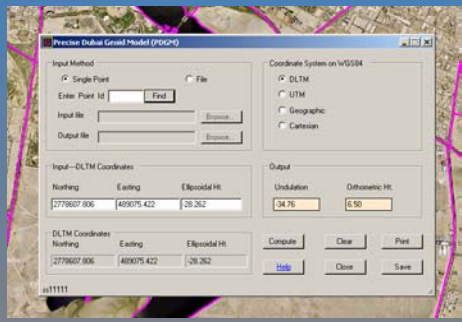
Precise Dubai Geoid Model - Program



Developed by Survey Section, DM

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CONCLUSION

- The derived Geoid model is precise enough to meet the Third order levelling Standards.
- In the main land Dubai the accuracy could be of the order of 1-3 cm on average
- In Hatta region the accuracy could be of 5-10 cm. Still more information and data is necessary to improvise the model in this region.
- We believe this Geoid model could meet the requirement of many potential users who would intend to convert GPS heights into their corresponding Mean Sea Level heights.

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