Development of a Geoid-Ellipsoid Separations Model in Israel

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Key words: orthometric heights**,** ellipsoidal heights**,** geoid separations

SUMMARY

The state network of geodetic stations by means of the GPS measurements was created in Israel in 1990-2000. These geodetic stations were connected with the acting permanent network (APN) and high-grade GPS stations. Geographic coordinates of 7 APN stations in datum ITRF-2000 were used for the network adjustment. The ellipsoidal heights resulting from this adjustment are applied in the given model.

Since the beginning of the project, the task of this network application was formulated so as to enable the network to determine the values of orthometric heights by means of the GPS measurements. Therefore the stations were created so that they would either coincide with the benchmarks or would be easily nivelled in the Datum of Israel orthometric heights. All such stations are expected to have both ellipsoidal and orthometric heights. Ellipsoid-geoid differences range from 16 to 24 meters on the territory of Israel.

The model was developed on the basis of 684 points with the given ellipsoidal and orthometric heights. Geostatistical method of Kriging approximation was used for the construction of the undulation surface. The values of differences were calculated on the net 0.5 x 0.5 km. To define the accuracy of the undulation calculations the Kriging standard deviations values were calculated on the same net Undulation accuracy ranges from 3 to 15 cm and depends on the network density and the accuracy of the ellipsoidal heights.

Orthometric heights in the given model are assumed to be free of error. To prove the validity of the model, a series of measurements of ellipsoidal heights was conducted in 2004-5 in different parts of the country. The measurements were taken on 96 benchmarks which were not included into the model. Orthometric heights estimated by means of the model demonstrated good agreement of the model with the orthometric heights datum. The obtained residuals almost always agree with the estimations computed by means of the model. The discrepancy does not exceed 0.14 and relative deviation values of residuals from the permitted by the model do not exceed 0.34.

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1. INTRODUCTION

The State Network of the GPS stations is being developed in Israel since 1990. In 1994 the problem was put forward to define the orthometric heights by means of the GPS measurements which are much cheaper and require less time than construction of the nivelling loops especially for long distances.

To solve this problem, geoid-ellipsoid separations need to be calculated in every point of the surface where coordinate measurements are performed. It becomes feasible under the following conditions:

- a reliable connection is set up between the GPS and nivelling networks;
- the exact geoid model is constructed;
- tools are created to estimate geoid-ellipsoid separations in every point of the surface.

The solution of this problem is described in (Forai, Papo, Sharni, 1998) for the Carmel Ridge area.

This paper describes the development of a geometrical model in which a mathematical surface formed by the points of the State nivelling network is used instead of the gravimetric geoid.

2. DATA USED FOR THE DEVELOPMENT OF THE MODEL.

2.1. GPS points network.

The GPS points network consists of:

- 18 constantly operating stations, in Israel called Active Permanent Network (APN)
- 155 high precision stations (G1) connected to APN;
- about 800 (U) of 3-4 class precision stations;

New U-stations were designed to measure both a position and a height, therefore they were mounted so as to be included into the State nivelling network. Wherever possible, GPS measurements were also performed on the existing benchmarks which were included into the U-stations network. In 1994-1997 the U network existed in fragments which were used separately, some of the points being added later. In 1999 U-network fragments were united and connected with 26 points of G1 and 7 APN stations acting at that moment. As a result U-network was adjusted and geographic coordinates of the stations in WGS84 were obtained.

The following are the characteristics of the network:
- 794 U stations out of the total of 895;
- 2343 GPS vectors processed;
- 101 APN and G1 permanent stations fixed;

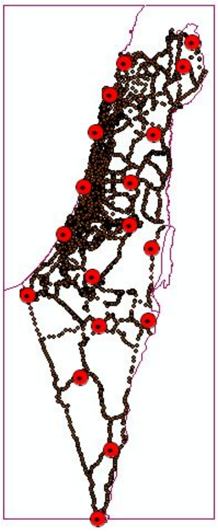

Fig1.GPS points network

The following data refer to the ellipsoidal heights:
- mean square error is 0.023m;
- standard deviation is 0.018m;
- 34 points with the error exceeding 0.05m, in 13 of them an error exceeds 0.06m;

The maximal error refers to the stations in the Dead Sea area.

2.2 State Nivelling Network

State nivelling network was being created during a long period of time. At present it consists of the stations of 5 classes of precision. The first class network consists of 855 points combined into 15 loops. The height values were calculated for the stations as a result of the network adjustment with one fixed station connected to a mariograph.

The second class network consists of 1588 stations combined into the loops based on the first class height values. The third class network is based on the second and first class stations. The fourth and the fifth class stations are far less precise, therefore they are used mainly in engineering applications.

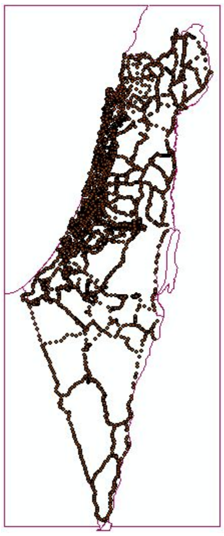

Fig 2. The State Nivelling Network

The latest adjustment of the state heights network was performed in 1998 preceded by the thorough data analysis. In order to achieve maximal consistency, stable repeated height differences were adjusted wherever possible. Maximal consistency is particularly important to obtain because the network was being created during a long period of time

3. CONSTRUCTION OF THE GEOID –ELLIPSOID SEPARATIONS SURFACE

To develop the undulation model, the first, second and third class points were used. As an exception, in the Negev and Arava deserts and the Dead Sea area, where the higher precision points have not been created yet, the fourth-class points were used, which undoubtedly influenced the precision if the model.
It proved possible to construct the surface on the basis of 684 GPS stations of the U class, which are simultaneously used as the benchmarks.

3.1 Using Kriging Interpolation

To interpolate geoid-ellipsoid separations, in the present model Kriging interpolation (optimal prediction) method is applied. Together with calculation of the approximate function value, it is important to estimate the approximation error. In order to do it, additional information about the properties of the function (e.g. derivatives, extremums, etc.) is required. This information is difficult to specify if the values of the function are obtained empirically and distorted by measurement errors. When the (geo) statistical methods are used, one always has to deal with the following problem: how to apply the statistical method in the case of only one-time realization of the random process. To deal with the problem, numerous spatial realizations (moved at the distance S from each other) of the same set are used.

Suppose that, after the removal of the trend, the separation function $U=U(x, y)$ satisfies the geostatistic intrinsic hypothesis. Because the function is smooth and changes slowly, (within the zone of at least 15 km), changes, indeed, depend rather on the distance between the points

than on their coordinates. Nowadays, application of the geostatistical methods has become an integral part of the software such as ArcGIS or Golden Software.
Thus, the unknown value of $U_0 = U(x_0, y_0)$ is calculated as weighted linear combination of neighboring values of U_0.

$$U_0 = \sum_{i=1}^{n} W_i * U_i \qquad \sum_{i=1}^{n} W_i = 1 \tag{1}$$

The error is: $r_0 = U_0 - \sum_{i=1}^{n} W_i * U_i$
and the error variance is:

$$\sigma_{R_0}^2 = \sigma^2 + \sum\sum W_i W_j \operatorname{cov}(U_i, U_j) - 2\sum W_i \operatorname{cov}(U_i, U_0) + 2\mu(\sum W_i - 1) \qquad \sigma^2 = \operatorname{cov}(U_0, U_0) \tag{2}$$

μ is the Lagrange parameter.
Weights W_i are defined under the condition of the minimum error variance which leads to the system of equations:

$$\sum W_j \operatorname{cov}(U_i, U_j) + \mu = \operatorname{cov}(U_i, U_0) \tag{3}$$

The ordinary Kriging method implies that, instead of covariance for the weights calculations, variogram is used.

Kriging standard deviation is taken as an interpolation error value. A number of numerical experiments were conducted to determine the variogram parameters in order to obtain close-to-real values of the interpolated function and interpolation errors. (Isaacs,Srivastava,1989)

3.2 Some Technical Details

Separations grid was built in 3 steps: the first resolution being 4 x 4 km, the second 1 x 1 km and the final 0.5 x 0.5 km.
Scattered data interpolation for all the steps was:

- search radius (km) 218
- number of sectors to search 4
- maximum number of data to use from each sector 6
- minimum number of data in all sectors 5

In the areas close to the borders the data were available in two or even one sectors only.
Even in these cases at least 5 numbers were taken for the interpolation.

Parameters of variograms:

- experimental:

	step 1	step 2	step 3
max. lag distance(km) :	140	10	10
number of lags	25	25	25
lag width (km)	5.6	1.0	0.4

- model:

nugget effect	0	0	0
function	linear	linear	linear
slope (xE-0.5)	1.98	0.5	0.1
anisotropy:			
ratio	1	1	2
angle(deg.)	0	0	121.1

	step 1	step 2	step 3
grid resolution(km)	4x4	1x1	0.5x0.5

3.3. Realization of the Model

Separation values on the territory of Israel vary from 16 to 24 meters. Separation accuracy varies from 3.5cm to 15cm. As expected, the highest accuracy is achieved in the central and northern parts of the country where the distance between the known 684 points is 3 to 5 km. The least reliable separation is obtained in the Negev and Arava deserts and the area of the Dead Sea. See Fig. 3, Fig. 4 and Fig 5 for details.

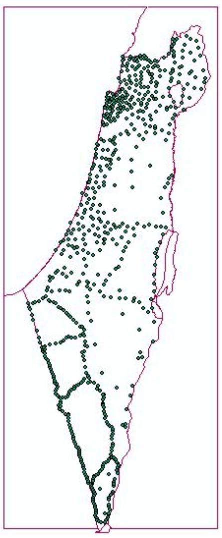

Fig 3. The 684 points of the model

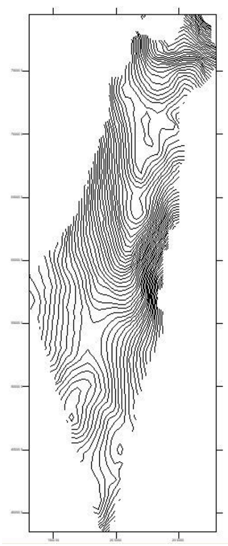

Fig 4. The isolines of the geoid-ellipsoid separations

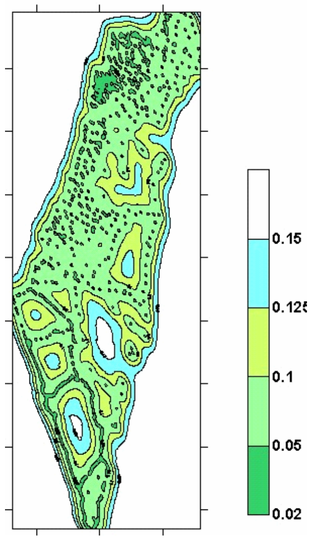

Fig 5. The isolines of the model accuracy values

In practice the model consists of two files: the file of the separation values and the file of errors on the 0.5 x 0.5 km grid (see Fig 6.).

$$und_0 = \sum_{i=1}^{i=4} \frac{und_i * 1/r_i^3}{\sum_{i=1}^{i=4} 1/r_i^3}$$

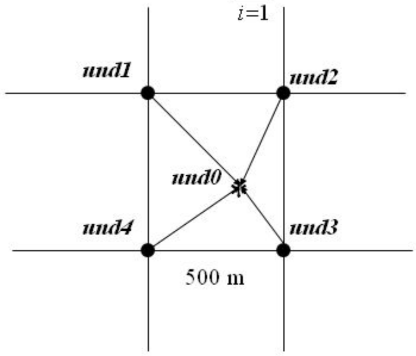

Fig 6. Calculation of the arbitrary value with 0.5 x 0.5 km grid

Errors of the separation values are computed similarly.
The program of computing the separation value in an arbitrary point $U(X_0, Y_0)$ runs in two regimes:

- interactive, as a calculator;
- file reading and processing.

See Fig. 7

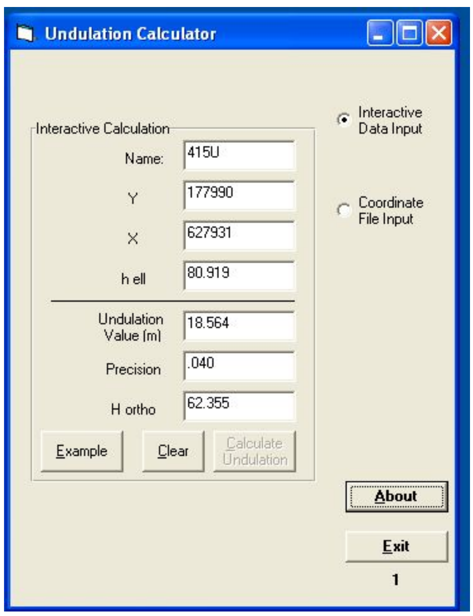

Fig 7. The Undulation Calculator Program

4. MODEL VALIDATION BY MEASUREMENTS.

The validity of the model was verified by means of the field measurements. During the years 2004-05 a special series of the field measurements was performed. They were either GPS measurements on the benchmarks or nivelling on the GPS stations. Neither the benchmarks nor the GPS stations were previously included into the model. The stations were placed in the points that varied by the predicted accuracy; it could be the north or the center of the country, the mountains or the desert region. In almost every case a good agreement with the model was observed. Discrepancies do not exceed 0.14 and relative deviation values of the residuals from those predicted by the model do not exceed 0.34.

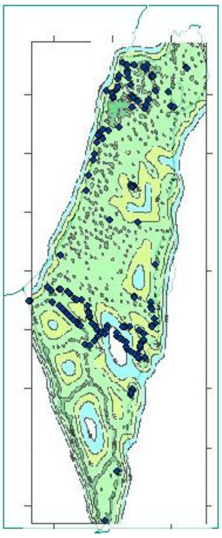

Fig 8. Field measurements proving the model validity

Since the end of 2005, re-measuring of the GPS stations, nivelling of those without orthometric heights and construction of the new stations instead of the destroyed ones have been performed. The new stations are provided with both GPS and nivelling measurements. All the stations, new and old, are being verified by the model. Thus, about 200 stations have already been verified. In most cases the efficiency of the model has been demonstrated. In several cases subsidence of stations was detected. A decision was taken to test each fluctuation in the field to make sure that the measurements are correct so that the model would be corrected accordingly

After all the field work has been completed, a model will be developed with the new values of the geoid- ellipsoid separations in the known points. The number of the known points will be increased which will result in the increased accuracy of the model.

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BIOGRAPHICAL NOTES

Academic experience: Applied mathematics MSc Novosibirsk State University, the USSR, 1971
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Lecturer, senior lecturer, associated Professor, Novosibirsk Institute of Surveying and Mapping, the USSR (1971 – 1990)
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