A New Mathematical Approach to Cadastral Documents Processing for Parcel Boundaries Restoration

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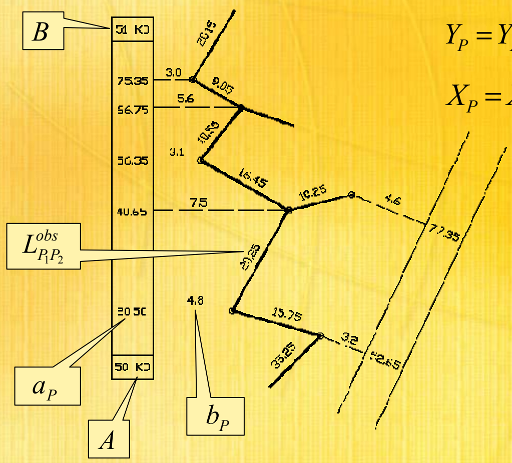
INTRODUCTION

- The Israeli cadastre:
  - 15,000 registered blocks
  - 700,000 parcels
- The customary method of ground surveying:
  - the orthogonal method
- The main cadastral documents:
  - special field books containing ground surveying details
  - official cadastral block maps containing parcels boundaries

ORTHOGONAL METHOD

Chain Surveying



Processing

$$Y_P = Y_A + a_P * (Y_B - Y_A) / L^{obs}_{AB} + b_P * (X_B - X_A) / L^{obs}_{AB}$$

$$X_P = X_A + a_P * (X_B - X_A) / L^{obs}_{AB} - b_P * (Y_B - Y_A) / L^{obs}_{AB}$$

$$\sigma_P{}^2 = \left(\frac{\partial F_P}{\partial a_P}\right)^2 \sigma_{a_P}{}^2 + \left(\frac{\partial F_P}{\partial b_P}\right)^2 \sigma_{b_P}{}^2$$

$$L^{calc}_{P_1P_2} = \sqrt{(Y_{P_2} - Y_{P_1})^2 + (X_{P_2} - X_{P_1})^2}$$

$$L^{calc}_{P_1P_2} \neq L^{obs}_{P_1P_2}$$

MAIN DRAWBACKS

Lack of possibility

- to consider redundant observations
- to provide optimal solution to large discrepancies:
  - between measured and calculated fronts
  - between registered and calculated parcels areas
- to apply geometrical and cadastral constraints

PROPOSED METHOD

- Cadastral documents processing by means of Least Squares Adjustment of redundant observations
- Applying geometrical and cadastral constraints to parcel turning points position

NON-CONSTRAINED SOLUTION

- Functional relation between observations and Y unknown parameters β

  $Y = F(\beta)$
- The Gauss-Markov adjustment model

  $\hat{Y} = X\hat{\beta} + \hat{\varepsilon}$
- Linearization of the nonlinear adjustment process

  $$Y_b - F(\beta_o) = \frac{\partial F(\beta_o)}{\partial \beta_o} * d\beta + \dots$$
- Iterative solution

  $$d\hat{\beta} = (X'PX)^{-1} X'PY \longrightarrow \hat{\beta}_a = \hat{\beta}_o + d\hat{\beta}$$

CONSTRAINED SOLUTION (1)

- Functional constraints

$$G(\beta) = 0$$

- Constraints linearization

$$G(\beta_o) + \frac{\partial G(\beta_o)}{\partial \beta_o} * d\beta + \ldots = 0$$

- The model of simultaneous adjustment

$$\begin{bmatrix} X^T PX & H^T \\ H & P_c^{-1} \end{bmatrix} * \begin{bmatrix} d\beta \\ k \end{bmatrix} = \begin{bmatrix} X^T Py \\ w \end{bmatrix}$$

CONSTRAINED SOLUTION (2)

- Geometrical (implicit) constraints identification

$$F = \frac{1}{m} * (\hat{\beta}_c - \hat{\beta})^T * \Sigma_\beta^{-1} * (\hat{\beta}_c - \hat{\beta})$$

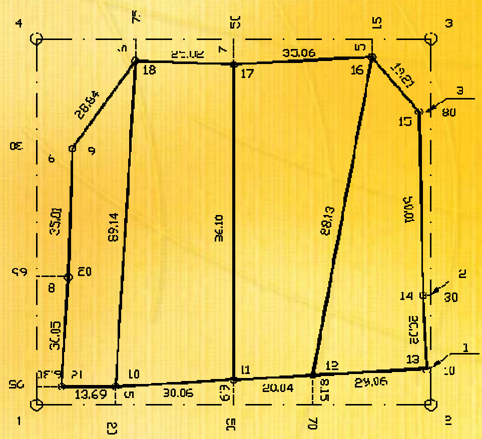
$\hat{\beta}_c$ - point coordinates calculated with constraints
$\hat{\beta}$ - point coordinates calculated without constraints
Σ_β - point positional variance calculated without constraints

- Hypothesis test (statistical F-test)

$H_0 : \hat{\beta}_c = \hat{\beta}$ vs. $H_1 : \hat{\beta}_c \neq \hat{\beta}$ **(null hypothesis vs. alternative hypothesis)**

If $F < F_{\alpha,m,n-m}$ - $\hat{\beta}_c$ may me accepted

SIMULATION – Idea



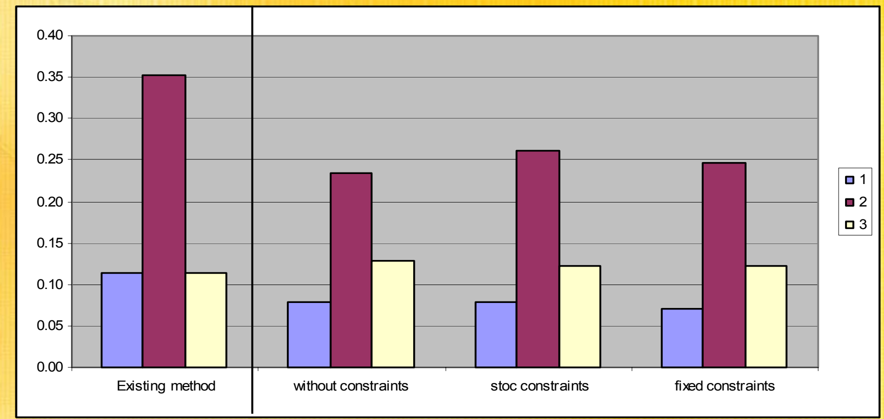
1. The source: “true” original turning points coordinates and “true observations” fitting completely the geometry of the sample
2. “Spoiling” of “true observations” by applying a normally distributed errors
3. Processing according to the existing and proposed methods

SIMULATION - Details

- 3 situations of observations “spoiling”
    1. Assumed observations accuracy
    2. Low observations accuracy
    3. Large discrepancies between calculated and measured fronts
- Proposed adjustment method application
    1. Observations adjustment without constraints
    2. Stochastic constraints application
    3. Fixed constraints application
- Constraints
    1. Points co-linearity
    2. Lines parallelism

SIMULATION – Processing Results (1)

Point position error ellipses

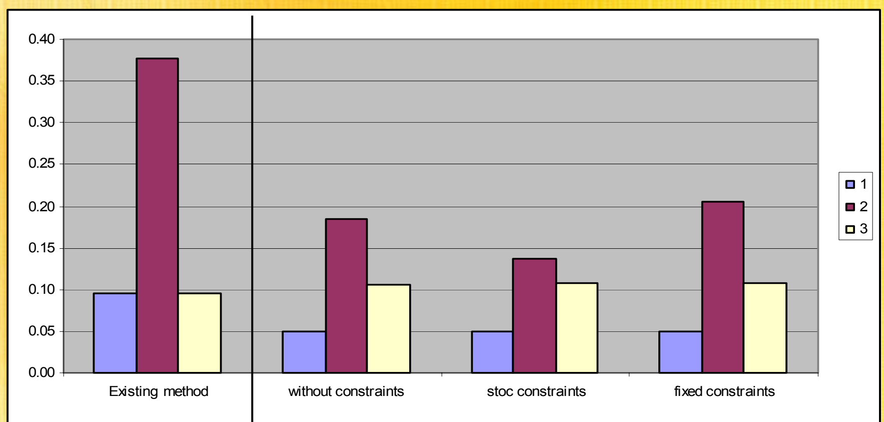


3 situations:

- assumed observations accuracy
- low observations accuracy
- large discrepancies between calculated and measured fronts

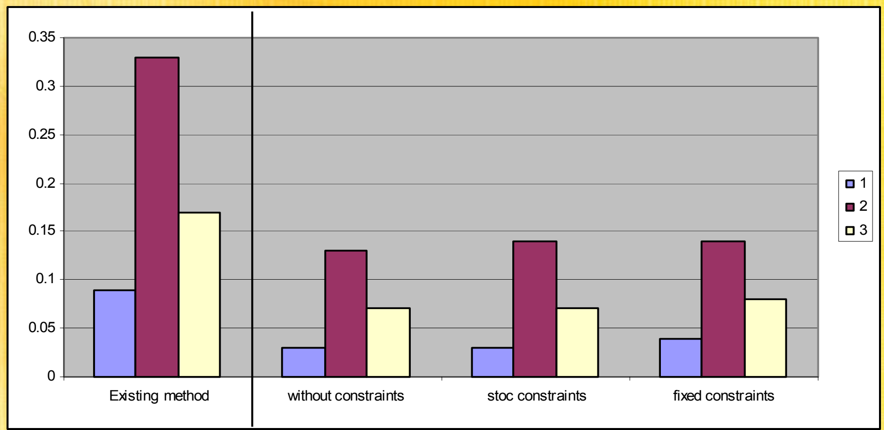
SIMULATION – Processing Results (2)

Differences between the adjusted points coordinates and the "true" coordinates



SIMULATION – Processing Results (3)

Observations mean residuals



SIMULATION – Comparison

Proposed method vs. existing one:

- In situations (1) and (2) [assumed & low accuracy]
  - points positional errors are reduced
  - points coordinates are obtained closer to their "true" position
- In all three situations
  - observations residuals are considerably reduced

SIMULATION – F-test

Implementation:

- Additional constraint, not existing in the original situation, has been imposed

Results:

Calculated $F_{0.05,24,15}$	Proposed method with stochastic constraints	Proposed method with fixed constraints	Tabular F_{calc}	Test
Without additional constraints	0.425	0.655	2.288	Accepted
With additional constraints	8.637	19.602	2.288	Denied

- Constraints existing in the original situation, are accepted; not existing – are denied

PROPOSED METHOD ADVANTAGES

- Optimal coordination between different kinds of cadastral information
- Keeping ground observations, having juridical validity, maximal closeness to their adjusted values
- Increasing positional accuracy of parcels turning points
- Forcing parcels to retain their original geometrical form

FUTURE WORK

- Analysis of the weights of the original ground observations (achieving “correct” weights)
- Analysis of “correct/true” constraints weights and order of their implementation
- Connection of separate blocks of parcels to a homogeneous seamless cadastral space

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Thank You