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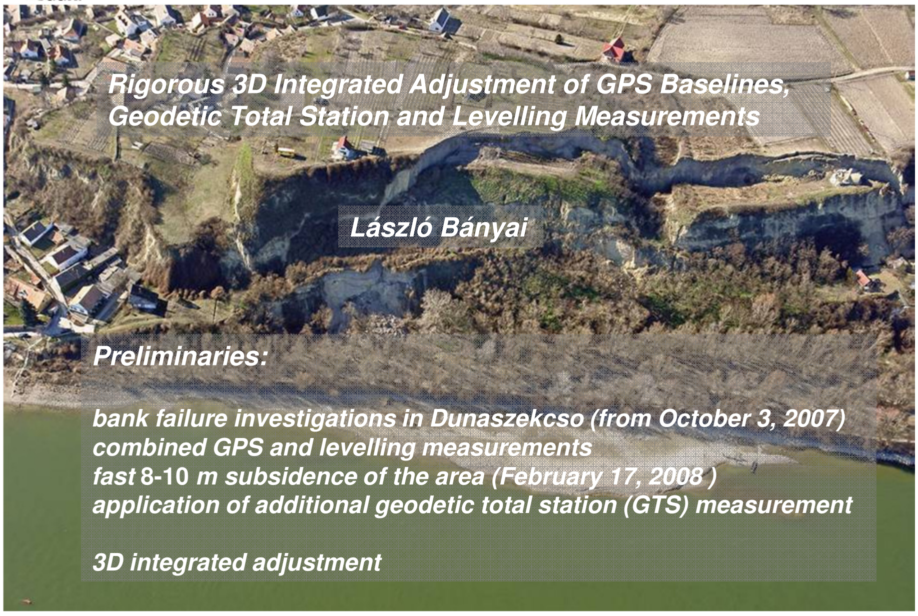

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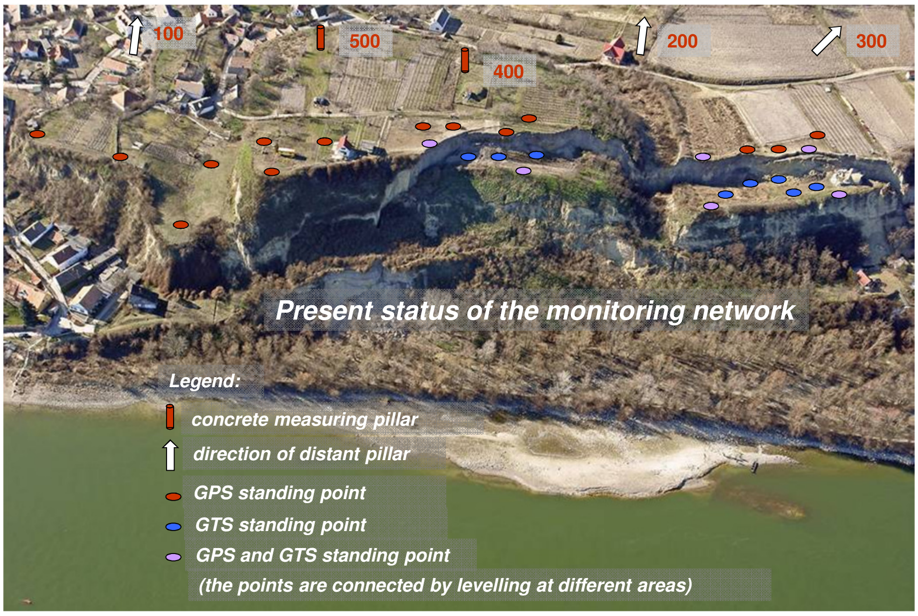

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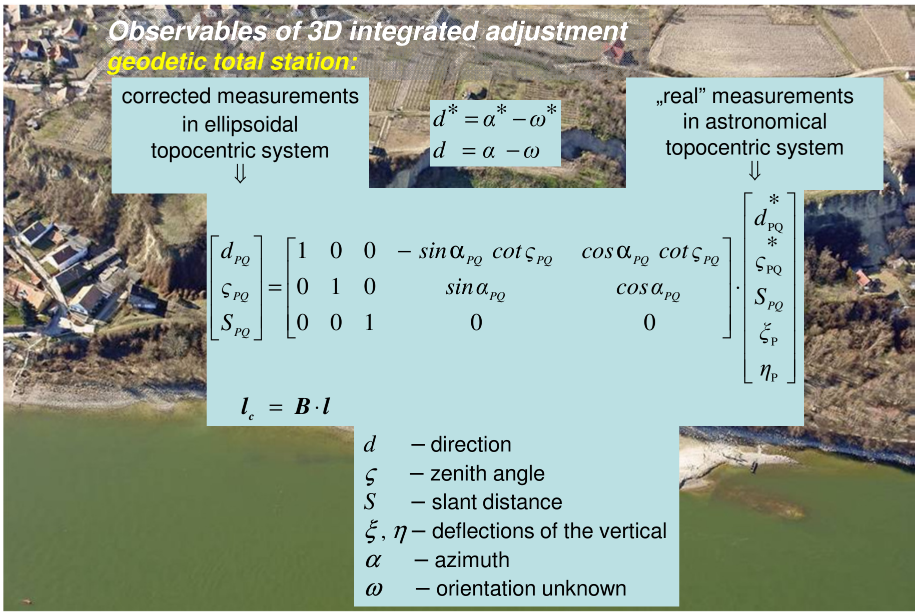

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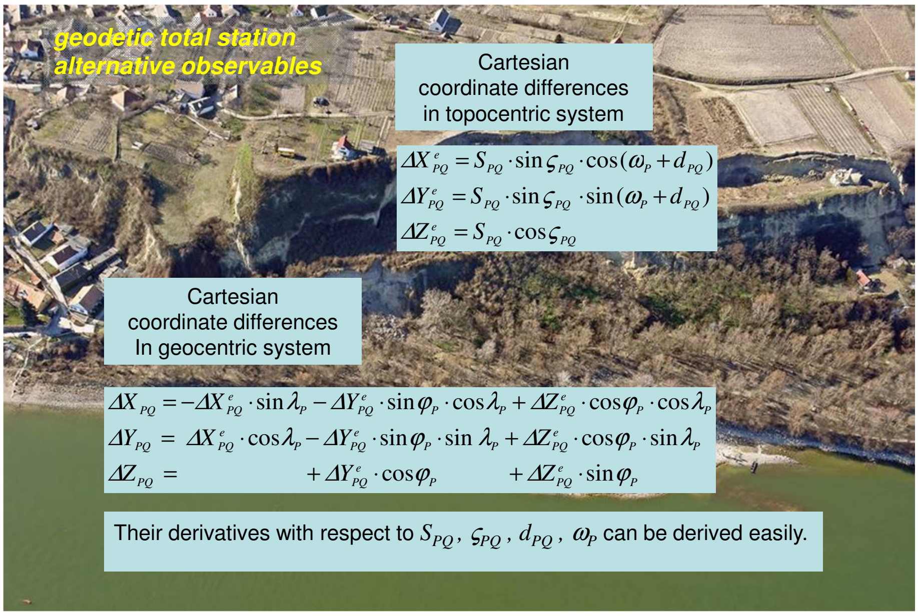

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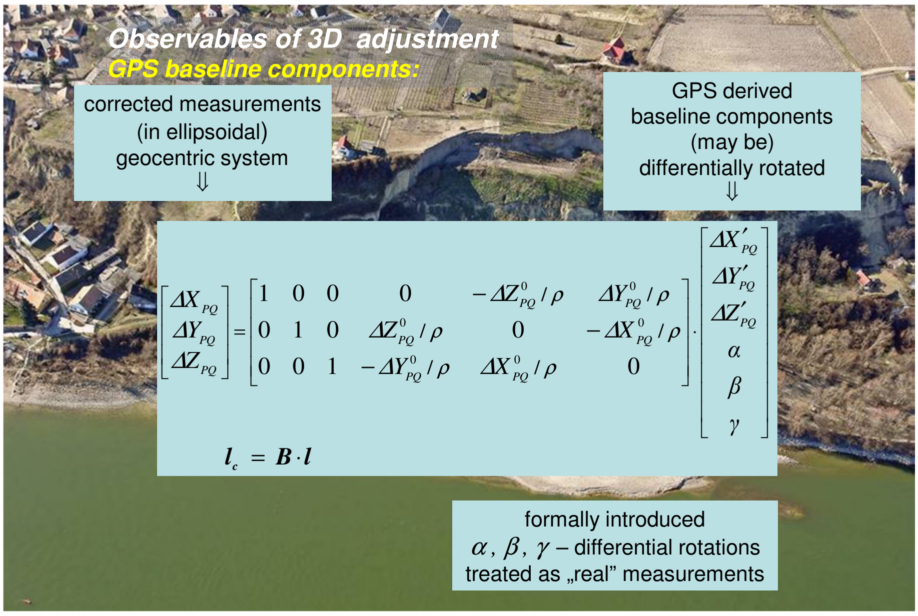

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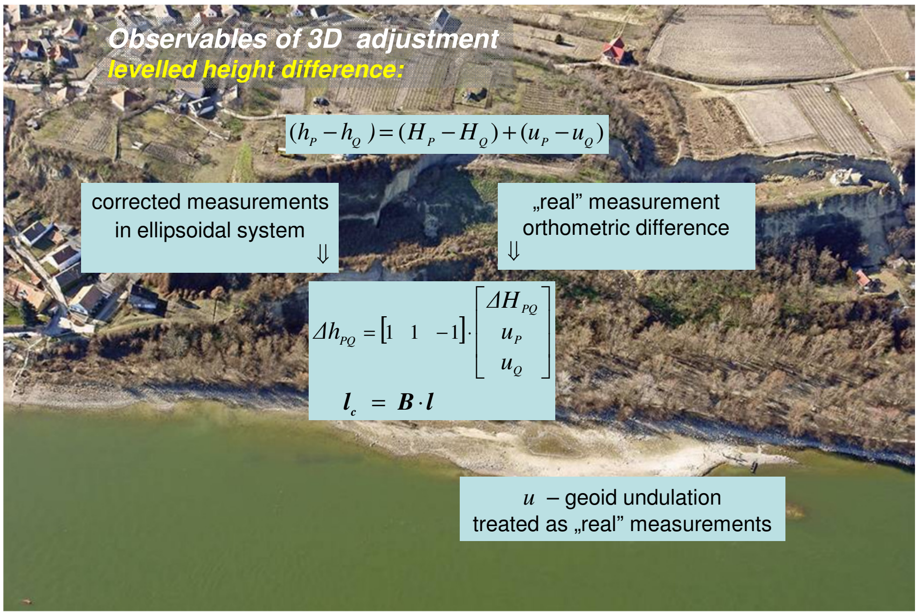

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Adjustment models (Mikhail 1967)

adjustment of observations and functionally independent parameters:

$$
\begin{aligned}
Bv &= Ax - b \\
P &= Q^{-1} \\
P_c &= (BQB^t)^{-1} \\
\hat{x} &= (A^t P_c A)^{-1}(A^t P_c b) \\
\hat{v} &= Q B^t P_c (A\hat{x} - b) \\
Q_{\hat{x}\hat{x}} &= (A^t P_c A)^{-1} = N^{-1} \\
Q_{\hat{v}\hat{v}} &= Q B^t (P_c - P_c A N^{-1} A^t P_c) B Q \\
Q_{\hat{l}\hat{l}} &= Q - Q_{\hat{v}\hat{v}} \\
\hat{\sigma}_0^2 &= \frac{v^t P v}{m-n}
\end{aligned}
$$

adjustment of the indirect observations:

$$
\begin{aligned}
v_c &= Ax - b \\
Q_c &= BQB^t \\
P_c &= Q_c^{-1} \\
\hat{x} &= (A^t P_c A)^{-1}(A^t P_c b) \\
\hat{v}_c &= A\hat{x} - b \\
Q_{\hat{x}\hat{x}} &= (A^t P_c A)^{-1} = N^{-1} \\
Q_{\hat{v}_c\hat{v}_c} &= Q_c - A N^{-1} A^t \\
Q_{\hat{l}_c\hat{l}_c} &= Q_c - Q_{\hat{v}_c\hat{v}_c} \\
\hat{\sigma}_0^2 &= \frac{v_c^t P v_c}{m-n}
\end{aligned}
$$

- v – residual of „real" measurements
- v_c – residual of corrected measurements
- x – correction of the preliminary parameters
- Q , P – weight coefficient and weight matrices

Observation equation

geodetic total station (traditional approach):

$$v_c = Ax - b$$

$$
\begin{bmatrix} v_d \\ v_\varsigma \\ v_S \end{bmatrix} =
\begin{bmatrix}
\partial d_{PQ}/\partial X_Q & \partial d_{PQ}/\partial Y_Q & \partial d_{PQ}/\partial Z_Q & \partial d_{PQ}/\partial X_P & \partial d_{PQ}/\partial Y_P & \partial d_{PQ}/\partial Y_P & -1 \\
\partial \varsigma_{PQ}/\partial X_Q & \partial \varsigma_{PQ}/\partial Y_Q & \partial \varsigma_{PQ}/\partial Z_Q & \partial \varsigma_{PQ}/\partial X_P & \partial \varsigma_{PQ}/\partial Y_P & \partial \varsigma_{PQ}/\partial Z_P & 0 \\
\partial S_{PQ}/\partial X_Q & \partial S_{PQ}/\partial Y_Q & \partial S_{PQ}/\partial Z_Q & \partial S_{PQ}/\partial X_P & \partial S_{PQ}/\partial Y_P & \partial S_{PQ}/\partial Z_P & 0
\end{bmatrix} \cdot
\begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ x_P \\ y_P \\ z_P \\ \omega_P \end{bmatrix}
- \begin{bmatrix} d_{PQ} - d_{PQ}^0 \\ \varsigma_{PQ} - \varsigma_{PQ}^0 \\ S_{PQ} - S_{PQ}^0 \end{bmatrix}
$$

Derivatives are given in:
Leick (1995)
Strang & Borre (1997)

This approach is very sensitive to the preliminary coordinates !

$$Q = \langle \sigma_d^2/\sigma_0^2 , \sigma_\varsigma^2/\sigma_0^2 , \sigma_s^2/\sigma_0^2 , \sigma_\xi^2/\sigma_0^2 , \sigma_\eta^2/\sigma_0^2 \rangle$$

$$Q_c = BQB^t$$

σ_0 – a priori standard deviation of unit weight

Observation equation

$$\mathbf{v}_c = \mathbf{A}\,\mathbf{x} - \mathbf{b}$$

geodetic total station (alternative approach):

$$\begin{bmatrix} v_{\Delta X} \\ v_{\Delta Y} \\ v_{\Delta Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & \partial \Delta X_{PQ} / \partial \omega_P \\ 0 & 1 & 0 & 0 & -1 & 0 & \partial \Delta Y_{PQ} / \partial \omega_P \\ 0 & 0 & 1 & 0 & 0 & -1 & \partial \Delta Z_{PQ} / \partial \omega_P \end{bmatrix} \cdot \begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ x_P \\ y_P \\ z_P \\ \omega_P \end{bmatrix} - \begin{bmatrix} \Delta X_{PQ} - X_Q^0 + X_P^0 - m_Q \cdot \cos\varphi_Q^0 \cdot \cos\lambda_Q^0 + m_P \cdot \cos\varphi_P^0 \cdot \cos\lambda_P^0 \\ \Delta Y_{PQ} - Y_Q^0 + Y_P^0 - m_Q \cdot \cos\varphi_Q^0 \cdot \sin\lambda_Q^0 + m_P \cdot \cos\varphi_P^0 \cdot \sin\lambda_P^0 \\ \Delta Z_{PQ} - Z_Q^0 + Z_P^0 - m_Q \cdot \sin\varphi_Q^0 + m_P \cdot \sin\varphi_P^0 \end{bmatrix}$$

„quasy" linear observation equations

m – standing height of instrument and prism

$$\mathbf{Q} = \left\langle \sigma_d^2/\sigma_0^2 , \sigma_\varsigma^2/\sigma_0^2 , \sigma_s^2/\sigma_0^2 , \sigma_\xi^2/\sigma_0^2 , \sigma_\eta^2/\sigma_0^2 \right\rangle$$

σ_0 – a priori standard deviation of unit weight

$$\mathbf{Q}_c = \mathbf{D}\,\mathbf{B}\,\mathbf{Q}\,\mathbf{B}^t\,\mathbf{D}^t$$

$\mathbf{D}$ – derivatives with respect to d , ς , S measurements

Observation equation

$$\mathbf{v}_c = \mathbf{A}\,\mathbf{x} - \mathbf{b}$$

GPS baseline components:

$$\begin{bmatrix} v_{\Delta X} \\ v_{\Delta Y} \\ v_{\Delta Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ x_P \\ y_P \\ z_P \end{bmatrix} - \begin{bmatrix} \Delta X_{PQ} - (X_Q^0 - X_P^0) \\ \Delta Y_{PQ} - (Y_Q^0 - Y_P^0) \\ \Delta Z_{PQ} - (Z_Q^0 - Z_P^0) \end{bmatrix}$$

linear observation equations

$$\mathbf{Q} = \left\langle \sigma_\alpha^2 , \sigma_\beta^2 , \sigma_\gamma^2 \right\rangle$$

$$\mathbf{Q}_c = \frac{1}{\sigma_0^2}\mathbf{B} \begin{bmatrix} s^2\mathbf{M} & 0 \\ 0 & \mathbf{Q} \end{bmatrix} \mathbf{B}^t$$

$\mathbf{M}$ – GPS derived variance-covariance matrix
s – arbitrary variance scale factor

Observable equation

levelled height difference:

$$v_c = A\,x - b$$

$$[v_{\Delta h}] = \begin{bmatrix} \partial\Delta h_{PQ}/\partial X_Q & \partial\Delta h_{PQ}/\partial Y_Q & \partial\Delta h_{PQ}/\partial Z_Q \\ \partial\Delta h_{PQ}/\partial X_P & \partial\Delta h_{PQ}/\partial Y_P & \partial\Delta h_{PQ}/\partial Z_P \end{bmatrix} \cdot \begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ x_P \\ y_P \\ z_P \end{bmatrix} - \left[\Delta H_{PQ} - (h_P^0 - h_Q^0) - (u_P - u_Q)\right]$$

$$Q = \left\langle \sigma^2_{\Delta H}/\sigma_0^2\ ,\ \sigma^2_{u_P}/\sigma_0^2\ ,\ \sigma^2_{u_Q}/\sigma_0^2 \right\rangle$$

$$Q_c = \left[\frac{\sigma^2_{\Delta H} + \sigma^2_{u_Q} + \sigma^2_{u_P}}{\sigma_0^2}\right]$$

Derivatives are given in:
Strang & Borre (1997)
(spherical approach)

The adjustment procedure

Treatment of gravity field data (ξ, η, u):

can be
- neglected,
- used as corrections only,
- handled as zero quantities with known standard deviations
- handled as „real" measurements (estimated by proper model together with their standard deviation).

GPS baseline rotations (α, β, γ):

can be
- neglected,
- handled as zero quantities with known standard deviations

Estimation of GPS variance scale factor:

$s = \dfrac{\hat{\sigma}_0}{\sigma_0}$ derived only from GPS baseline adjustment at the first step

Datum definition:
- coordinates of selected group of banchmarks can be fixed, or
- their coordinate changes can be minimised (free network)

Practical application in Dunacsekcső

σ_S = 1 mm + 1 ppm (Leica TC2003 total station)

$\sigma_d = \sigma_\varsigma$ (equvivalent with σ_S in the distance of the prism)

$\xi = \eta = 0$, $\sigma_\xi = \sigma_\eta = 1''$

u (is neglected – small area)

s = 10 – 25 (experianced GPS variance scale)

σ_{dH} = 0.15 - 0.30 mm

σ_0 = 0.001

Practical experiances:

– „quasy" linear observation equations works with pour preliminary coordinates, too

– levelling significantly improve the precision of height components

– everage coordinate precision 0.2-0.5 mm