

FIG Working Week, Marrakech, Marokko, May 18-22 2011

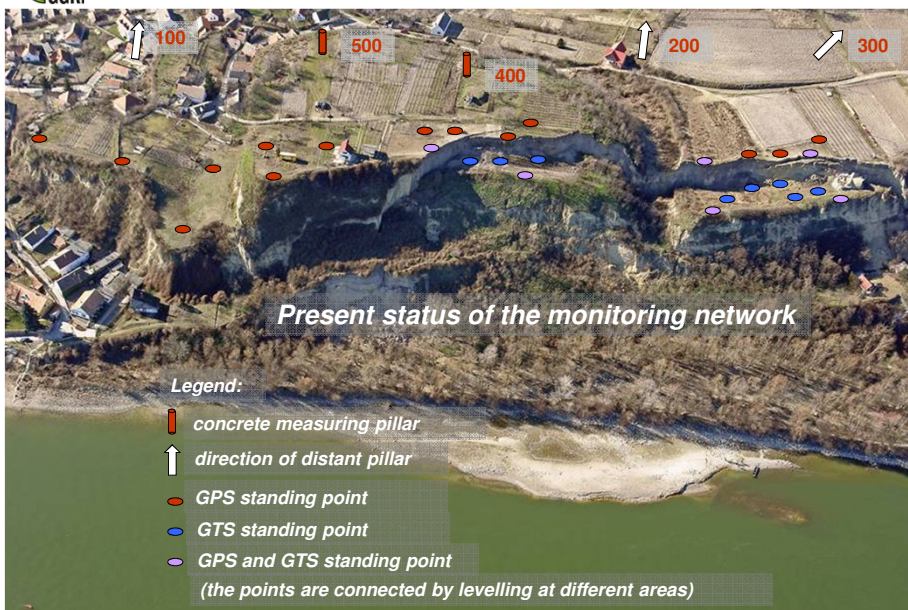


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Observables of 3D integrated adjustment
geodetic total station:

corrected measurements
in ellipsoidal
topocentric system

$$d^* = \alpha^* - \omega^*$$

$$d = \alpha - \omega$$

„real” measurements
in astronomical
topocentric system

$$\begin{bmatrix} d_{PQ} \\ \zeta_{PQ} \\ S_{PQ} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\sin \alpha_{PQ} \cot \zeta_{PQ} & \cos \alpha_{PQ} \cot \zeta_{PQ} \\ 0 & 1 & 0 & \sin \alpha_{PQ} & \cos \alpha_{PQ} \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_{PQ}^* \\ \zeta_{PQ}^* \\ S_{PQ} \\ \zeta_P \\ \eta_P \end{bmatrix}$$

$$l_c = B \cdot l$$

- d – direction
- ζ – zenith angle
- S – slant distance
- ζ, η – deflections of the vertical
- α – azimuth
- ω – orientation unknown

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geodetic total station
alternative observables

Cartesian
coordinate differences
in topocentric system

$$\Delta X_{PQ}^e = S_{PQ} \cdot \sin \zeta_{PQ} \cdot \cos(\omega_p + d_{PQ})$$

$$\Delta Y_{PQ}^e = S_{PQ} \cdot \sin \zeta_{PQ} \cdot \sin(\omega_p + d_{PQ})$$

$$\Delta Z_{PQ}^e = S_{PQ} \cdot \cos \zeta_{PQ}$$

Cartesian
coordinate differences
in geocentric system

$$\Delta X_{PQ}^c = -\Delta X_{PQ}^e \cdot \sin \lambda_p - \Delta Y_{PQ}^e \cdot \sin \varphi_p \cdot \cos \lambda_p + \Delta Z_{PQ}^e \cdot \cos \varphi_p \cdot \cos \lambda_p$$

$$\Delta Y_{PQ}^c = \Delta X_{PQ}^e \cdot \cos \lambda_p - \Delta Y_{PQ}^e \cdot \sin \varphi_p \cdot \sin \lambda_p + \Delta Z_{PQ}^e \cdot \cos \varphi_p \cdot \sin \lambda_p$$

$$\Delta Z_{PQ}^c = \Delta Y_{PQ}^e \cdot \cos \varphi_p + \Delta Z_{PQ}^e \cdot \sin \varphi_p$$

Their derivatives with respect to $S_{PQ}, \zeta_{PQ}, d_{PQ}, \omega_p$ can be derived easily.

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Observables of 3D adjustment

GPS baseline components:

corrected measurements
(in ellipsoidal)
geocentric system

GPS derived
baseline components
(may be)
differentially rotated

$$\begin{bmatrix} \Delta X_{PQ} \\ \Delta Y_{PQ} \\ \Delta Z_{PQ} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -\Delta Z_{PQ}^0 / \rho & \Delta Y_{PQ}^0 / \rho \\ 0 & 1 & 0 & \Delta Z_{PQ}^0 / \rho & 0 & -\Delta X_{PQ}^0 / \rho \\ 0 & 0 & 1 & -\Delta Y_{PQ}^0 / \rho & \Delta X_{PQ}^0 / \rho & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta X'_{PQ} \\ \Delta Y'_{PQ} \\ \Delta Z'_{PQ} \\ \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$l_c = B \cdot l$$

formally introduced
 α, β, γ – differential rotations
treated as „real” measurements

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Observables of 3D adjustment

levelled height difference:

$$(h_p - h_Q) = (H_p - H_Q) + (u_p - u_Q)$$

corrected measurements
in ellipsoidal system

„real” measurement
orthometric difference

$$\Delta h_{PQ} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \Delta H_{PQ} \\ u_p \\ u_Q \end{bmatrix}$$

$$l_c = B \cdot l$$

u – geoid undulation
treated as „real” measurements

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Adjustment models (Mikhail 1967)

adjustment of observations and functionally independent parameters:

$$\begin{aligned}
 Bv &= Ax - b \\
 P &= Q^{-1} \\
 P_c &= (BQB')^{-1} \\
 \hat{x} &= (A'P_cA)^{-1}(A'P_cb) \\
 \hat{v} &= QB'P_c(A\hat{x} - b) \\
 Q_{\hat{x}\hat{x}} &= (A'P_cA)^{-1} = N^{-1} \\
 Q_{\hat{v}\hat{v}} &= QB'(P_c - P_cAN^{-1}A'P_c)BQ \\
 Q_{\hat{v}\hat{x}} &= Q - Q_{\hat{v}\hat{v}} \\
 \hat{\sigma}_0^2 &= \frac{v'Pv}{m-n}
 \end{aligned}$$

adjustment of the indirect observations:

$$\begin{aligned}
 v_c &= Ax - b \\
 Q_c &= BQB' \\
 P_c &= Q_c^{-1} \\
 \hat{x} &= (A'P_cA)^{-1}(A'P_cb) \\
 \hat{v}_c &= A\hat{x} - b \\
 Q_{\hat{x}\hat{x}} &= (A'P_cA)^{-1} = N^{-1} \\
 Q_{\hat{v}_c\hat{v}_c} &= Q_c - AN^{-1}A' \\
 Q_{\hat{v}_c\hat{x}} &= Q_c - Q_{\hat{v}_c\hat{v}_c} \\
 \hat{\sigma}_0^2 &= \frac{v_c'Pv_c}{m-n}
 \end{aligned}$$

- v – residual of „real” measurements
- v_c – residual of corrected measurements
- x – correction of the preliminary parameters
- Q, P – weight coefficient and weight matrices

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Observation equation geodetic total station (traditional approach):

$$v_c = Ax - b$$

$$\begin{bmatrix} v_d \\ v_s \end{bmatrix} = \begin{bmatrix} \partial d_{PQ} / \partial X_Q & \partial d_{PQ} / \partial Y_Q & \partial d_{PQ} / \partial Z_Q & \partial d_{PQ} / \partial X_P & \partial d_{PQ} / \partial Y_P & \partial d_{PQ} / \partial Z_P & -1 \\ \partial \zeta_{PQ} / \partial X_Q & \partial \zeta_{PQ} / \partial Y_Q & \partial \zeta_{PQ} / \partial Z_Q & \partial \zeta_{PQ} / \partial X_P & \partial \zeta_{PQ} / \partial Y_P & \partial \zeta_{PQ} / \partial Z_P & 0 \\ \partial S_{PQ} / \partial X_Q & \partial S_{PQ} / \partial Y_Q & \partial S_{PQ} / \partial Z_Q & \partial S_{PQ} / \partial X_P & \partial S_{PQ} / \partial Y_P & \partial S_{PQ} / \partial Z_P & 0 \end{bmatrix} \cdot \begin{bmatrix} X_Q \\ Y_Q \\ Z_Q \\ X_P \\ Y_P \\ Z_P \\ \omega_P \end{bmatrix}$$

Derivatives are given in:

Leick (1995)

Strang & Borre (1997)

This approach is very sensitive to the preliminary coordinates !

$$\begin{bmatrix} d_{PQ} - d_{PQ}^0 \\ \zeta_{PQ} - \zeta_{PQ}^0 \\ S_{PQ} - S_{PQ}^0 \end{bmatrix}$$

$$Q = \langle \sigma_d^2 / \sigma_0^2, \sigma_\zeta^2 / \sigma_0^2, \sigma_s^2 / \sigma_0^2, \sigma_x^2 / \sigma_0^2, \sigma_y^2 / \sigma_0^2, \sigma_z^2 / \sigma_0^2 \rangle$$

$$Q_c = BQB'$$

σ_0 – a priori standard deviation of unit weight

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Observation equation

geodetic total station (alternative approach):

$$v_c = A x - b$$

$$\begin{bmatrix} v_{\Delta X} \\ v_{\Delta Y} \\ v_{\Delta Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & \frac{\partial \Delta X_{PQ}}{\partial \omega_p} \\ 0 & 1 & 0 & 0 & -1 & 0 & \frac{\partial \Delta Y_{PQ}}{\partial \omega_p} \\ 0 & 0 & 1 & 0 & 0 & -1 & \frac{\partial \Delta Z_{PQ}}{\partial \omega_p} \end{bmatrix} \cdot \begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ x_P \\ y_P \\ z_P \\ \omega_p \end{bmatrix} - \begin{bmatrix} \Delta X_{PQ} - X_Q^0 + X_P^0 - m_Q \cdot \cos \phi_Q^0 \cdot \cos \lambda_Q^0 + m_P \cdot \cos \phi_P^0 \cdot \cos \lambda_P^0 \\ \Delta Y_{PQ} - Y_Q^0 + Y_P^0 - m_Q \cdot \cos \phi_Q^0 \cdot \sin \lambda_Q^0 + m_P \cdot \cos \phi_P^0 \cdot \sin \lambda_P^0 \\ \Delta Z_{PQ} - Z_Q^0 + Z_P^0 - m_Q \cdot \sin \phi_Q^0 + m_P \cdot \sin \phi_P^0 \end{bmatrix}$$

„quasy” linear observation equations

m – standing height of instrument and prism

σ_0 – a priori standard deviation of unit weight

$$Q = \langle \sigma_d^2 / \sigma_0^2, \sigma_\zeta^2 / \sigma_0^2, \sigma_s^2 / \sigma_0^2, \sigma_\xi^2 / \sigma_0^2, \sigma_\eta^2 / \sigma_0^2 \rangle$$

$$Q_c = DBQB'D'$$

D – derivatives with respect to d, ζ, S measurements

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Observation equation

GPS baseline components:

$$v_c = A x - b$$

$$\begin{bmatrix} v_{\Delta X} \\ v_{\Delta Y} \\ v_{\Delta Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ x_P \\ y_P \\ z_P \end{bmatrix} - \begin{bmatrix} \Delta X_{PQ} - (X_Q^0 - X_P^0) \\ \Delta Y_{PQ} - (Y_Q^0 - Y_P^0) \\ \Delta Z_{PQ} - (Z_Q^0 - Z_P^0) \end{bmatrix}$$

linear observation equations

$$Q = \langle \sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2 \rangle$$

$$Q_c = \frac{1}{\sigma_0^2} B \begin{bmatrix} s^2 M & 0 \\ 0 & Q \end{bmatrix} B'$$

M – GPS derived variance-covariance matrix
 s – arbitrary variance scale factor

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Observable equation
levelled height difference:

$$v_c = A x - b$$

$$[v_{dh}] = \begin{bmatrix} \frac{\partial \Delta h_{PQ}}{\partial X_Q} & \frac{\partial \Delta h_{PQ}}{\partial Y_Q} & \frac{\partial \Delta h_{PQ}}{\partial Z_Q} \\ \frac{\partial \Delta h_{PQ}}{\partial X_P} & \frac{\partial \Delta h_{PQ}}{\partial Y_P} & \frac{\partial \Delta h_{PQ}}{\partial Z_P} \end{bmatrix} \cdot \begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ x_P \\ y_P \\ z_P \end{bmatrix} - \left[\Delta H_{PQ} - (h_P^0 - h_Q^0) - (u_P - u_Q) \right]$$

$$Q = \left(\sigma_{\Delta H}^2 / \sigma_0^2, \sigma_{u_P}^2 / \sigma_0^2, \sigma_{u_Q}^2 / \sigma_0^2 \right)$$

$$Q_c = \left[\frac{\sigma_{\Delta H}^2 + \sigma_{u_Q}^2 + \sigma_{u_P}^2}{\sigma_0^2} \right]$$

Derivatives are given in:
Strang & Borre (1997)
(spherical approach)

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The adjustment procedure

Treatment of gravity field data (ξ, η, u):

- can be – neglected,
- used as corrections only,
- handled as zero quantities with known standard deviations
- handled as „real” measurements (estimated by proper model together with their standard deviation).

GPS baseline rotations (α, β, γ):

- can be – neglected,
- handled as zero quantities with known standard deviations

Estimation of GPS variance scale factor:

$$s = \frac{\hat{\sigma}_0}{\sigma_0} \text{ derived only from GPS baseline adjustment at the first step}$$

Datum definition:

- coordinates of selected group of banchmarks can be fixed, or
- their coordinate changes can be minimised (free network)

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Practical application in Dunacsekcső

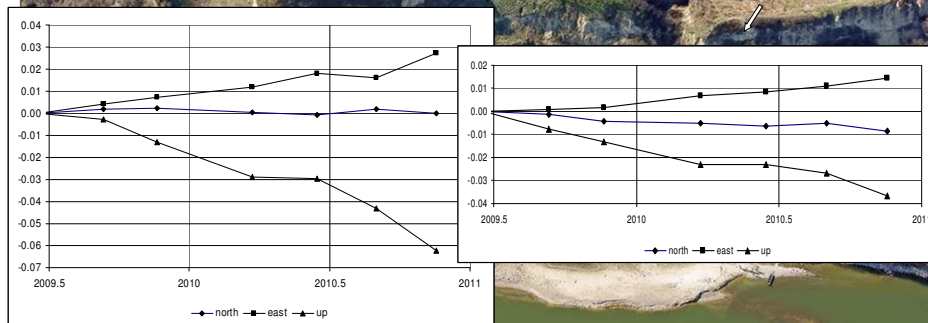
$\sigma_s = 1 \text{ mm} + 1 \text{ ppm}$ (Leica TC2003 total station)
 $\sigma_d = \sigma_s$ (equivalent with σ_s in the distance of the prism)
 $\xi = \eta = 0, \sigma_\xi = \sigma_\eta = 1''$
 u (is neglected – small area)
 $s = 10 - 25$ (experienced GPS variance scale)
 $\sigma_{dH} = 0.15 - 0.30 \text{ mm}$
 $\sigma_0 = 0.001$

Practical experiences:

- „quasy” linear observation equations works with poor preliminary coordinates, too
- levelling significantly improve the precision of height components
- average coordinate precision 0.2-0.5 mm

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Practical application in Dunacsekcső



Thank you for your attention !

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