DEVELOPMENT OF GEOID MODEL - A CASE STUDY ON WESTERN INDIA

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Key words: GNSS, GCP, Geoid, Gravity, Global Geo-potential Model

SUMMARY

The word 'height' has no singular meaning. Ideally, the physical meaningful height should refer to the Geoid which is an equipotential surface of the earth's gravity field and closely approximates the mean sea level in global sense neglecting the long term effect of Sea Surface Topography (SST). However, practically, it is very difficult to establish such a surface due to the complexities involved. For all developmental and engineering projects, Mean Sea Level (MSL) based heights are required and accordingly contours on Survey of India topographical maps are based on MSL based heights which fulfill the criteria of water flow. Therefore, in India and also most of the countries, heights are computed with respect to locally determined MSL and are provided by levelling work which is very laborious and also time consuming. The GNSS (Global Navigation Satellite System) technique has achieved remarkable improvement in horizontal as well as vertical positioning. However, the heights resulted from GNSS data processing are called ellipsoidal heights (h). It has created a pressing desire for a similarly fast and accurate determination of orthometric heights (H). The task of transforming the ellipsoidal height (h) obtained from GNSS technique to the orthometric height (H) has prompted geodesists around the world to determine the geoid model which allows to bridge GNSS-derived geometrical (Ellipsoidal) heights (h) to meaningful physical (Orthometric) heights (H) in their region of interest by the relation, $H = h^{GNSS} - N$. 'N' is known as geoid height or geoid undulation. Geoid modeling can be determined with basic input data of the acceleration of gravity near the Earth's surface (g).Survey of India has covered whole country with relatively dense gravity data. Survey of India did a case study to compute hybrid geoid model over Western India in a region from 20° to 31° in latitude and 71° to 79° in longitude. Firstly, gravimetric geoid was computed with terrestrial gravity, satellite altimetry derived marine gravity, SRTM terrain data and best fitted Global Geopotential Model (GGM) in the region using Remove-Compute-Restore technique that involves spherical Fast Fourier Transformation (FFT) with optimized Stokes's kernel. 14 cm RMSE accuracy was achieved in gravimetric geoid. It was subsequently fitted by LSC method with data of GNSS observations on 1st order bench marks at 39 locations and was tested at 13 such locations within the model area to yield hybrid geoid model wherein a RMSE of 7 cm has been achieved. Survey of India now plans to develop precise hybrid geoid model of entire country in near future.

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1. INTRODUCTION :

To address the challenges of real world, functional and meaningful heights are required. Ideally, the physical meaningful height should refer to the Geoid which is an equipotential surface of the earth's gravity field that closely approximates with MSL (Mean Sea Level) neglecting long term effect of sea surface topography (SST). However, practically, it is very difficult to establish such a surface due to the complexities involved. For all developmental and engineering projects, Mean Sea Level (MSL) based heights are required and accordingly contours on Survey of India topographical maps are based on MSL heights which fulfill the criteria of water flow. Therefore, in India and most of the countries, heights are computed with respect to MSL and are provided by spirit levelling work which is very laborious and also time consuming. After the advent of satellite based GNSS technique, positioning has become much easier than before. GNSS observations are based on a reference ellipsoid (like WGS84) which is different from Geoid (MSL). GNSS observed heights, which are referenced on reference ellipsoid are called 'ellipsoidal heights'. Ellipsoidal heights (h) cannot be used to determine where water will flow, and therefore are not used in topographic mapping as well as in developmental/engineering projects. An illustrative explanation is shown below:

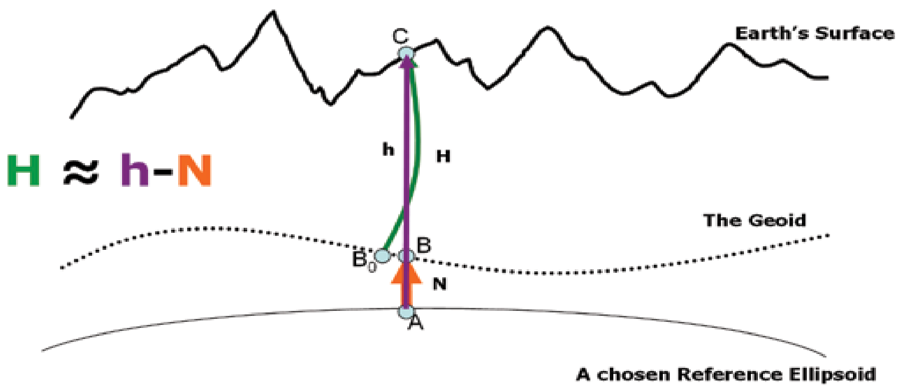

Fig. 1: Relation between Ellipsoid and Geoid ($H = h^{GNSS} – N$)

GNSS derived ellipsoidal heights require value of 'N' to get orthometric heights (H). The use of GNSS technology for determining fast and accurate ellipsoid heights has created a pressing desire for a similarly fast and accurate determination of orthometric heights. The task of transforming the ellipsoidal height (h) obtained from GNSS technique to the orthometric height has prompted geodesists around the world to determine the high precision geoid undulations (N), for their region of interest. During the last decade, the need for precise geoid model has gained momentum to minimize/eliminate the arduous & time consuming task of levelling. In order to transform from ellipsoid heights to orthometric heights, a model of the geoid must be computed, and geoid modeling can only be done with measurements of the acceleration of gravity near the Earth's surface (g). Many countries across the world have already developed their own geoid model which serve as the means of deriving orthometric heights from GNSS observations. These countries are also on the path of refining their geoid model to get more accurate orthometric heights.

2. GEOID COMPUTATION METHODS:

2.1. The geometric method:

2.1.1 astro-geodetic method:

It uses the direction of gravity vector employing geometrical technique. Geoid undulation (N) is determined by astronomical/natural coordinates (computed by star observations) and geodetic coordinates (computed by GNSS observations) of some stations. For this, deflection of vertical is determined first and then geoid undulation is computed. The deflection of vertical at any ground point is the angle between direction of gravity (plumb line) and the ellipsoid normal. This method has the limitations in terms of the requirement of an initial point/station within the region where geoid undulation($N_{\circ}$) is accurately known as only relative geoidal undulation(ΔN) are computed with respect to initial point.

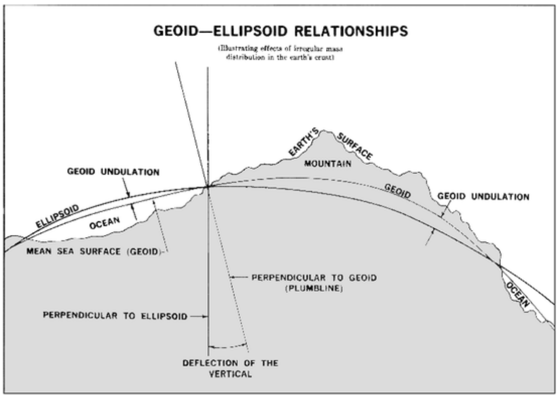

Fig. 2.: Deflection of vertical.

2.1.2 GNSSs/spirit levelling method:

GNSS observations in configuration with spirit levelling enables a direct estimation of the position of geoid of discrete points. Ellipsoidal height (h) from GNSS observation and orthometric height (H) from spirit levelling yields geoid undulation (N) from the relation $N=h^{GNSS} - H$. From the available values of N, closely fit geoid surface is created by using different degrees of polynomials. This method has its own limitations as laborious spirit levelling is required for the area of interest and also GNSS observations need to be carried out on levelling benchmarks. It is used for determination of the local geoid model in a relatively small area with an aim to replace leveling measurements with GPS surveys. It is normally used for engineering projects etc.

2.2 gravimetric method:

The geoid is essentially an equipotential surface of the Earth's gravity field that corresponds most closely with mean sea level (MSL) in the open oceans ignoring the effects of quasi-stationary sea surface topography. Theoretically, Stokes' formula is used for determination of gravimetric geoid which requires global coverage of gravity data/anomalies. The irresolvable constraint of having the global coverage of gravity data due to oceans & inaccessible areas has paved the way to evolve a new technique which includes global geopotential model (GGM) and digital elevation model (DEM) of the region alongwith some modification of Stokes' formula. This technique is called as Remove-Compute-Restore (RCR) method of geoid modeling. A modern gravimetric geoid uses a combination of three primary input data sources:

• a suitable global geopotential model, which provides most of the long and intermediate wavelength (>100 km) geoid undulations;

• terrestrial gravity data i.e. free air anomaly in and around the area of interest, which supply most of the intermediate wavelengths, and;

• a high-resolution digital elevation model (DEM), which supplies most of the short wavelengths, and is also required to satisfy theoretical demands of geoid computation from the geodetic boundary-value problem.

The pre-processing of these data is critically important, because if errors remain in any of these input data, they will directly propagate into the regional geoid model.

It is important to note that astro-geodetic methods use the direction of the gravity vector, employing geometrical techniques, whereas the gravimetric methods operates with the magnitude 'g', making use of potential theory.

2.3 Hybrid method:

It is a combination of geometrical and gravimetric method of geoid modelling. To fit the gravimetric geoid on to the terrain of study area, a set of well distributed GNSS/levelling data is used to model the corrector surface

3. STATUS OF MODEL/DATA & INFRASTRUCTURE:

3.1 earlier status:

In India, presently available nationwide geoid was computed a long time back and was created on the basis of lot of astro-geodetic observations with respect to Everest spheroid. It has various limitations and does not have any significance as far as GNSS solutions for orthometric height is concerned.

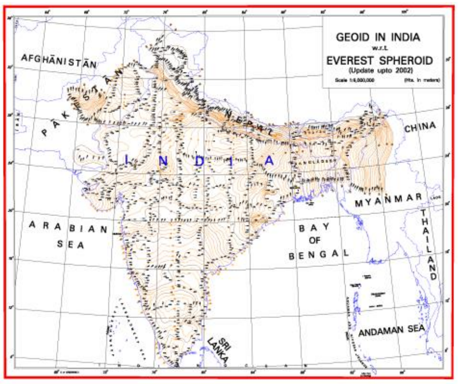

Fig. 3: Geoid undulation on Everest Spheroid

3.2 present status:

Survey of India (SoI) has the mandate to establish & maintain horizontal and vertical control network throughout India. Following is current status:

3.2.1 horizontal control network:

SoI has recently completed GCP library project wherein more than 2500 GNSS based reference stations on ITRF'2008 (epoch 2005) have been established throughout India.

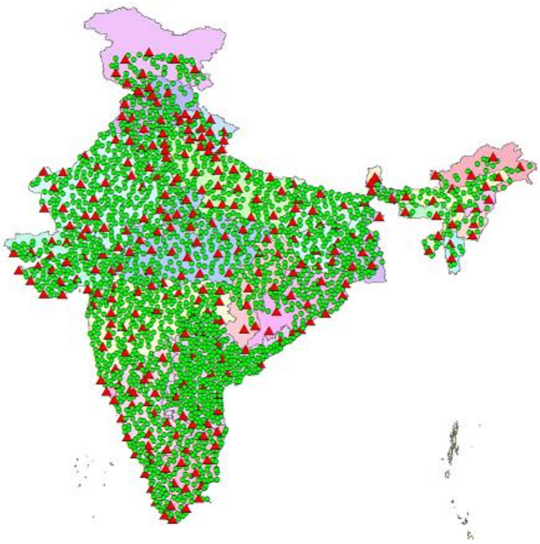

Fig.4: GCP Library Network

3.2.2 vertical control network:

High Precision Levelling data is also available in the entire country as shown below:

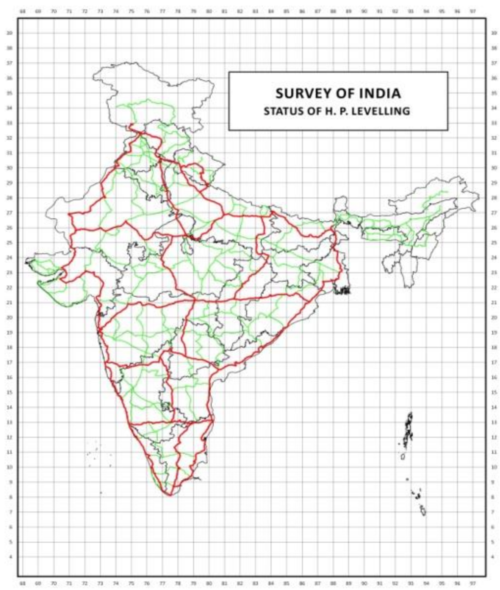

Fig.5: ‘Status of High Precision (1st order) levelling

3.2.3 gravity data in India:

Idea of determining the force of gravity(g) had been mooted in 1815 during execution of Great Triangulation Survey of India to account for deflection of vertical. In 1826, two pendulums were obtained from England, but no actual observations were taken with them. It was again revived in 1864 at the suggestion of the President of the Royal Society. Two brass pendulums were loaned by the Royal Society to the Govt. of India after calibration at the Kew Observatory in 1865 and was used by Basevi & Heaviside from 1865 to 1873, observations being made at about 30 stations from Cape Comorin to the More plain in Ladakh. Due to world wars, gravity observations were made intermittently from 1902 to 1923 with Sterneck's ½ sec. pendulums and from 1926 to 1939 with nickel-steel pendulum of Cambridge. However, broad frame-work of 564 gravity stations had become available by 1939. With a view to densify the stations for detailed studies, a Frost gravimeter and a Worden Geodetic Model gravimeter was purchased in 1947 and 1953 respectively and accordingly, gravity observations in 15 km mesh was thought of for whole country. Gravity has also been observed on high precision levelling lines & under different projects. Currently, large part of India has been covered with nearly 23,000 points of gravity data as shown below:

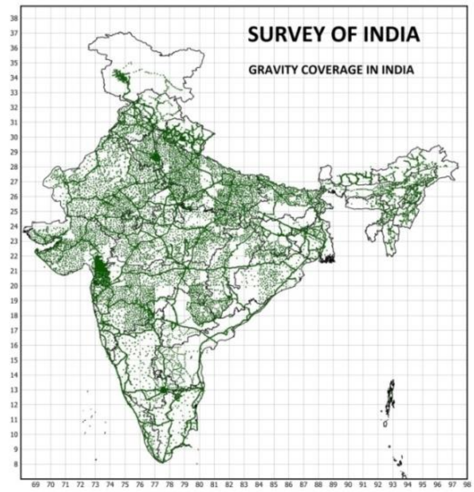

Fig. 6: Gravity Coverage

4. PILOT STUDY:

An area on western India has been selected for creating the geoid model from 20° to 31° in latitude and 71° to 79° in longitude.

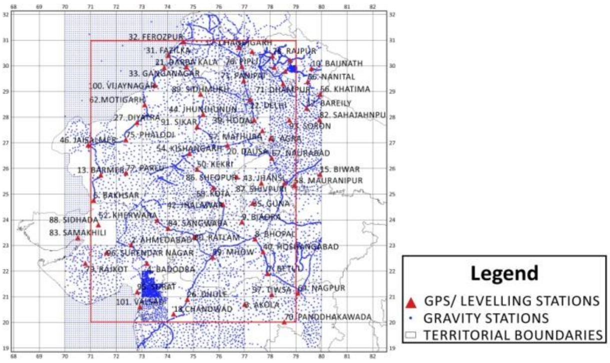

Fig. 7: Study Area

12 hours GNSS observations were done on the benchmarks (reference stations of MSL height) of high precision levelling within the area at suitable locations (as shown with red triangles in figure) for calculation of 'N'.

5. DATA PREPARATION:

5.1 Gravity Data:

SoI has been collecting the gravity data since more than 60 years. Therefore, data of study area was thoroughly scrutinized, cleaned & quality controlled to avoid errors for computation of free air anomaly (FA). Ellipsoidal height (h) was derived on newly implemented horizontal reference system based on ITRF'2008 (epoch 2005) to compute precise geoid undulation 'N' on those locations where GNSS observations were done on bench marks. SRTM (Shuttle Radar Topography Mission), an international project spearheaded by the U.S. National Geospatial-Intelligence Agency (NGA) and the U.S. National Aeronautics and Space Administration (NASA), global DEM has been used to compute the effects due to terrain. SRTM DEM data of 30" grid has been downloaded from website for this purpose.

5.2 Selection of suitable Global Geopotential model (GGM):

Free air anomaly (FA) & geoid undulation (N) of eight GGMs were downloaded in the grid of 0.25° X 0.25° from website of ICGEM (International Centre for Global Earth Models) which is one of five services coordinated by the International Gravity Field Service (IGFS) of the International Association of Geodesy (IAG). ICGEM has more than 100 GGMs & provides different functionals of the geopotential (e.g. geoid, gravity anomaly, gravity disturbance, equivalent water height) from a defined global model, on a specified grid and with respect to a defined reference system. Geoid undulation (N) of all six GGMs were compared with known 'N' of points within the area and RMSE calculated (as shown in Table 1) to finalize the suitable GGM which is closely fitted to our terrain of study. Degree of GGMs have been restricted to 720 as higher degree harmonic coefficients have negligible contribution to the value of geoid undulation 'N'.

Sl. No.	Global Gravity Model	Min	Max	Mean	RMSE
1	**XGM 2016 (degree 719)**	0.730	2.185	0.858	0.928
2	**GOCO05C (degree 720)**	0.808	2.232	0.846	0.924
3	**GGM05C (degree 360)**	**0.737**	**2.225**	**0.832**	**0.914**
4	**GECO (degree 720)**	0.699	2.246	0.859	0.931
5	**EIGEN6C4 (degree 720)**	0.696	2.260	0.871	0.939
6	**EIGEN6C3STAT (degree 720)**	0.678	2.227	0.879	0.947
7	**EGM08 (degree 720)**	0.828	2.248	0.894	0.968
8	**EGM08GOCE5 (degree 720)**	0.797	2.331	0.868	0.957

Table 1: Statistics of comparison of geoid undulation

GGM05C was selected as Global Geopotential Model for considering the effect of long wavelength in our model.

6. METHODOLOGY OF GRAVIMETRIC GEOID:

Purely gravimetric calculations of geoid heights is hampered by long wave systematic data errors and by inhomogeneous spatial resolutions and accuracy of the local gravity data. The global geopotential models generally provide the long wave part of the gravity field and dense local gravity data together with high resolution digital elevation model leads to a combined solution that can be applicable to a limited region, where data smoothening techniques are used by considering the terrain effect. A remove compute- restore technique (Tscherning, 1994) is applied in this study which includes the following steps.

Remove Steps: Gravity measurements data is subtracted with global gravity anomaly and surface correction.

a. Extract gravity anomaly from the spherical harmonic model to produce Δg_{ggm} (Long wavelength)

b. Extract gravity anomaly from SRTM data to produce $\Delta g_{terrain}$ (short-medium wavelength)

c. Subtract gravity anomalies Δg_{ggm} and $\Delta g_{terrain}$ from Δg_{FA}.

$$\Delta g_{res} = \Delta g_{FA} - \Delta g_{ggm} - \Delta g_{terrain}$$

Compute Steps: Residual gravimetric geoid is computed.

Residual gravity anomaly is arranged in a grid of 15′ x 15′ (corresponding to 27 x 27 km grid).

a. Apply spherical FFT (Fast Fourier Technique) with optimized Stoke's kernel to Δg_{res} to obtain geoid residual ΔN_{res}..

Restore Steps: Geoid residuals are summed with global geoid undulation and indirect effect, resulting to gravimetric geoid height (Undulation),

a. Compute global geoid undulation from GGM05C (selected GGM), ΔN_{ggm}.

b. Compute the terrain part $\Delta N_{terrain}$ from SRTM data.

c. Sum up ΔN_{res} (from remove steps), ΔN_{ggm} and $\Delta N_{terrain}$ to obtain $N_{gravimetric}$

$$N_{gravimetric} = \Delta N_{res} + \Delta N_{ggm} + \Delta N_{terrain}$$

A schematic diagram of the general computation procedure is given in following figure.

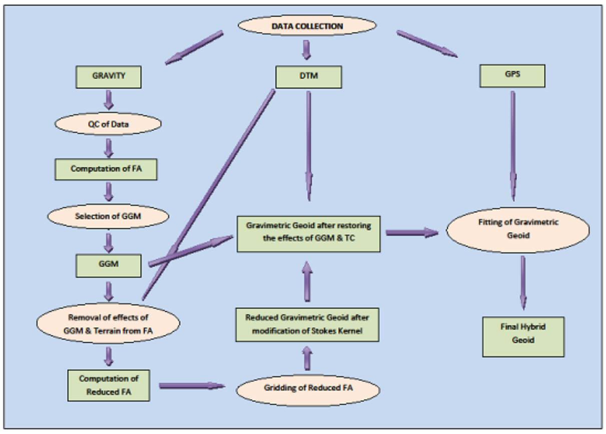

Fig. 8: Flow Chart of Development of Geoid Model

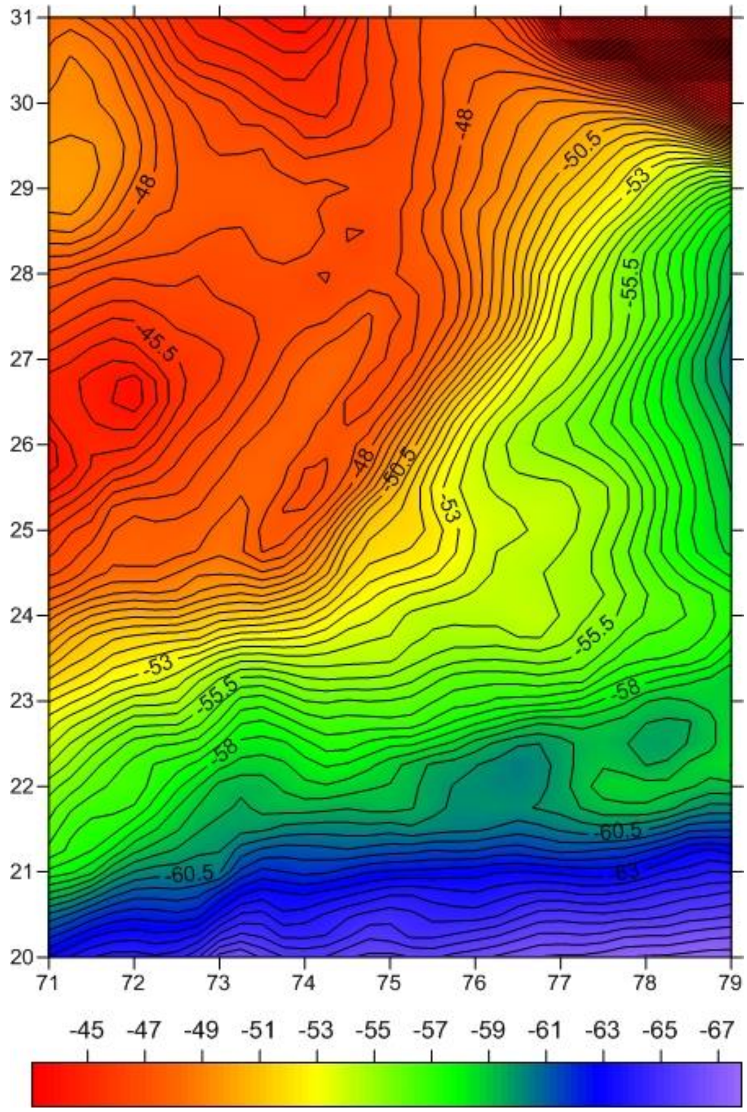

Fig. 9: Gravimetric Geoid

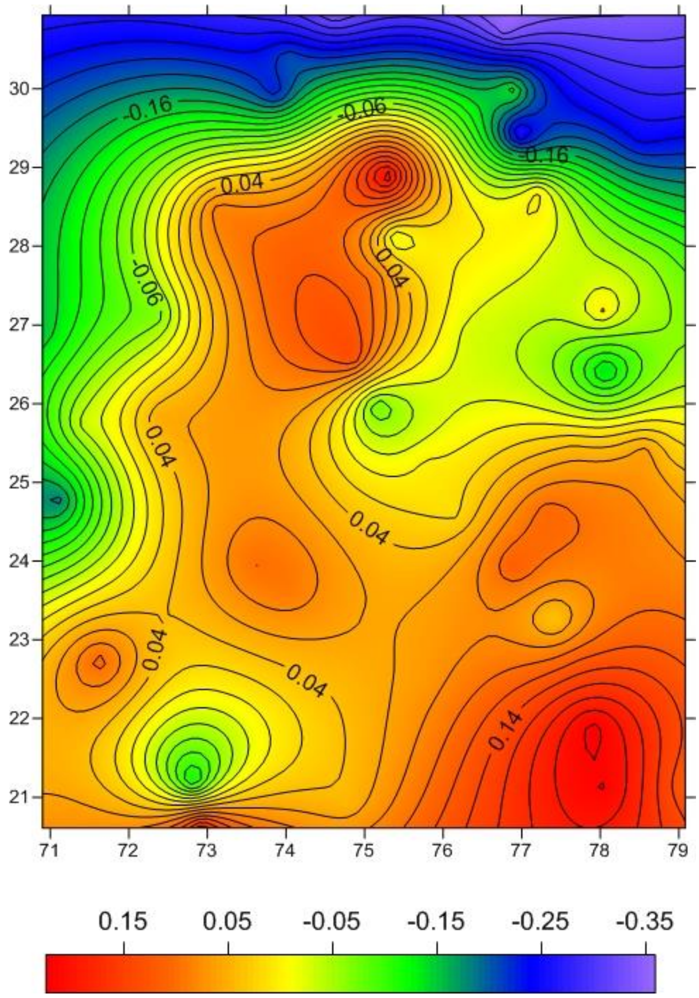

Fig. 10: Residual of Gravimetric & GNSSBM geoid heights

Minimum Difference	Maximum Difference	Mean	RMSE
-0.346	0.226	-0.005	0.136

Table 2: Statistics of residual of Gravimetric & GNSSBM geoid heights

7. MAKING OF HYBRID GEOID:

The outcome of the geoid computation method, based on gravity, give the gravimetric geoid - which in principle refers to a global reference system, i.e. global center of mass, average zero-potential surface. Generally, levelling zero or vertical datum of a country refers to local or regional mean sea-level, which is different from the global zero vertical datum due to the sea-surface topography. Users are interested in using GNSS to determine heights in a local vertical datum, to be consistent with existing levelling, there is a need to taylor the gravimetric geoid to the local level. The hybrid model is expected to improve the fit of gravimetric geoid to GNSS/levelling results. The hybrid geoid is determined by applying a corrector surface to gravimetric geoid. The development of corrector surface is basically the problem of modelling the residuals of GNSS/levelling (h-H) and gravimetric geoid heights ($N_{gravimetric}$) at co-located points. The gravimetric geoid is fitted with data of GNSS observations on 1st order bench marks (GNSSBM) at 39 locations by applying Least Square Collocation Technique to the residual data to yield hybrid geoid. Subsequently, hybrid geoid was validated at 13 GNSSBM locations within the model area and a RMSE of 7 cm has been achieved.

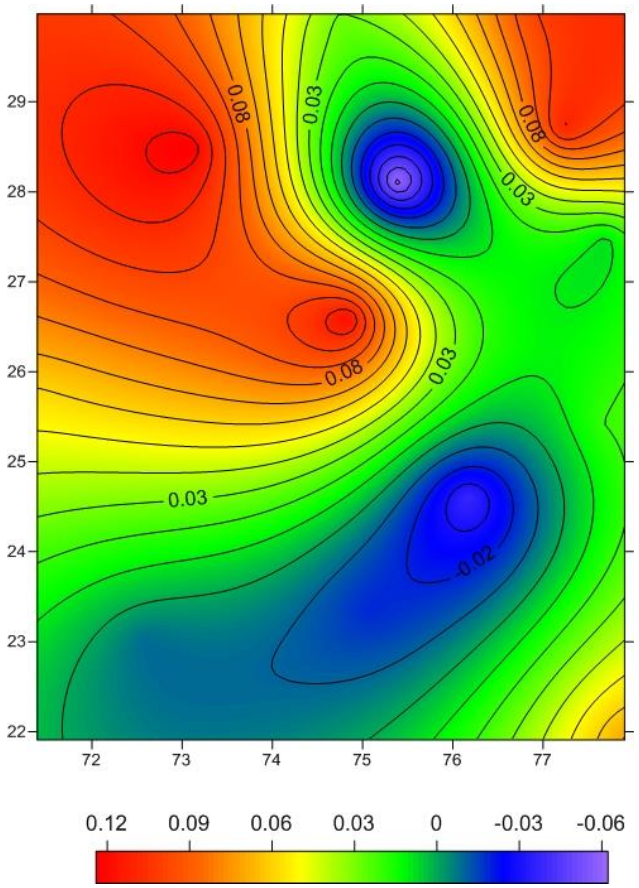

Fig. 11: Validation plot of hybrid geoid

Minimum Difference	Maximum Difference	Mean	RMSE
-0.064	0.125	0.039	0.072

Table 3: Statistics of hyrid geoid validation

8. CONCLUSION:

Geoid model is not necessary only for Geodetic applications to derive MSL heights directly by GNSS observations but it is also very useful for geo-physical applications & oceanographic study etc. It will pave the way for integration of island like Andaman etc. with vertical datum of main land. It will also facilitate the transformation from local vertical datum to global vertical datum. Additionally, Geoid Model will effectively contribute for revision/modernization of future vertical datum. Various kind of data collection i.e. field observed gravity, high precision levelling & GNSS observation on BM for development of precise Geoid Model of India is going on. However, Survey of India will soon release beta version Geoid Model of entire India.

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BIOGRAPHICAL NOTES:

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