Precise Determination of Hong Kong Geoid Using Heterogeneous Data

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Key words: Geoid determination, GPS/leveling, Gravity anomaly, Digital terrain model, Geopotential model.

ABSTRACT

The precise determination of local geoid is of considerable importance not only for surveying and mapping but also for studying some related problems of geophysics, oceanography and geodynamics, and now especially for applying GPS technique to determine orthometric or normal height in geodesy and surveying engineering. In the paper, based on the remove-restore technique, the Hong Kong gravimetric geoid with resolution of one kilometer has been computed using high degree global spherical harmonic geopotential model WDM94, gravity anomalies and digital terrain model which covering the whole territory of Hong Kong and its neighboring region. By comparing with precise GPS/leveling data the standard deviation of the geoid is ±7.6 cm, and the relative accuracy about 4ppm for baseline length longer than 50 km. The results illustrate that the systematic bias exists obviously between the two types of geoid heights. Employing four parameter transform model the gravimetric geoid can be improved significantly by precise GPS/leveling data. The results show that the standard deviation of the improved geoid is ±1.9 cm, and the relative accuracy better than 1ppm for baseline length longer than 40 km. Moreover, after the systematic bias removed the combined geoid was obtained by interpolating the residuals at grid points with Shepard surface fitting method, and the final geoid will be evaluated using independent precise GPS/leveling data in near future.

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1. INTRODUCTION

The geoid is loosely defined as the equipotential surface of the earth's gravity field at the mean sea level. It gives a consistent height system of topography on land as well as on sea. The precise determination of local geoid is of considerable importance not only for surveying and mapping but also for studying some related problems of geophysics, oceanography and geodynamics, and now especially for applying GPS technique to determine orthometric or normal height in geodesy and surveying engineering. Using EGM96 global spherical harmonic geopotential model, digital terrain model (DTM), high quality GPS/leveling data and gravity data on land and on sea which obtained at that time, the geoid of Hong Kong has been computed with remove-restore technique by Yang (1999), and he demonstrated that the accuracy of such derived geoid can arrive at about 2.6 centimeters, and that its relative accuracy better than 1ppm for baseline length longer than 40 kilometers. Since then more high precise GPS/leveling stations have been established in Hong Kong. Moreover, more gravity data and better DTM were collected. Therefore it's necessary to reconstruct the geoid of Hong Kong. In the paper all aspects involved in the practical computation and evaluation of the new geoid are described. The data sources and its preparation is first summarized. It is followed by discussing the selection of the best fitting geo-potential model, the fourth section discusses the method for the computation of gravimetric geoid height using Stokes formula with original spherical kernel function by 1-D fast Fourier transform (FFT) in the remove-restore approach. High quality GPS/leveling data should be used to remove the systematic biases of the gravimetric geoid through 4-parameter transformation model. In the fifth section the combined geoid is constructed with Shepard surface interpolation method.

2. DATA SOURCES

2.1 Digital terrain model

To compute terrain effects DTM of Hong Kong and its neighboring region must be constructed. 16 sheets of 1:20 000 digital topographical maps covering the whole territory of Hong Kong were used to generate DTMs with resolution 100m, 500m, and 1 km using software 3D Analyst of ArcView GIS version 3.1. 100m resolution DTM of Shenzhen was obtained. Figure 1 shows the topography of Hong Kong and its neighboring region of 70km by 70km. In the area the highest point is 823.0m above Hong Kong Principal Datum, the lowest –60.8m and the on average 73.7m.

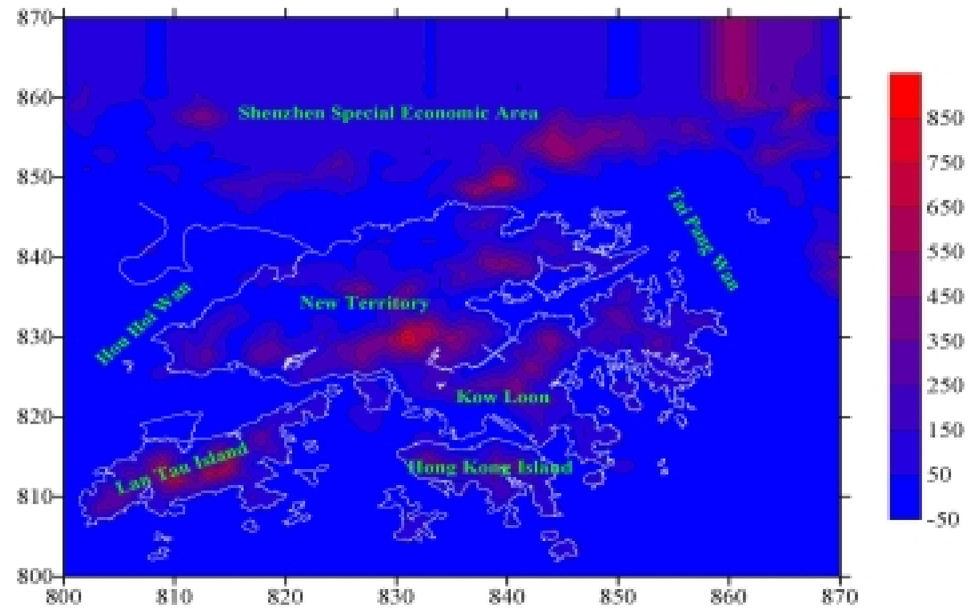

Figure 1 Topography of Hong Kong and its neighboring region at 1km grid (HK80 Grid Coordinates, Contour interval is 100m)

2.2 GPS/leveling data

There are 55 high quality GPS stations in Hong Kong. Their heights above the Hong Kong Principal Datum were determined with precise geometric leveling or trigonometric leveling. The accuracy specification of geometric leveling is $4\sqrt{k}$ ~ $8\sqrt{k}$ (mm) where k is the length of leveling routine in km. The estimated accuracy of leveled heights is 1~2 cm at 17 stations, 3~5 cm at 26 stations, and no better than 10 cm at 12 stations. The distribution of these GPS/leveling stations is displayed in Figure 2.

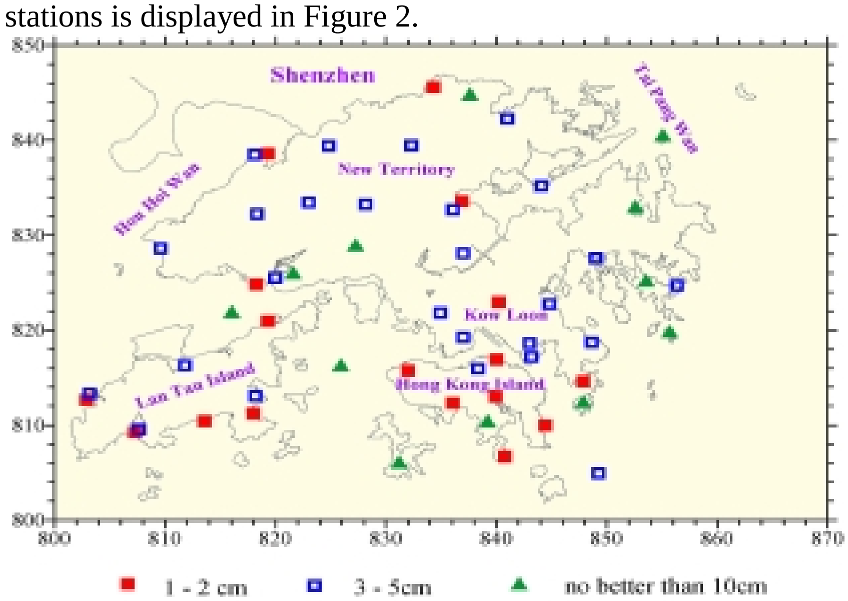

Figure 2 Distribution of the 55 GPS/leveling stations in Hong Kong (HK80 Grid Coordinates)

2.3 Gravity data

Hong Kong has 640 discrete gravity observations with station spacing 2 km on land and 2 4 km on sea. They were collected using Lacoste and Romberg model ‘G’ land gravity meter and model ‘H/U’ seabed gravity meter by Electronic and Geophysical Services Ltd. (EGS,1988; 1991). The local gravity base is connected to International Gravity Standardization Net 1971 (IGSN 71) with accuracy of 0.03 mGal (Evans, 1990). The gravity measurements in neighboring region Shenzhen are available: 3609 points on land and 1262 points on sea with 1 km resolution. They were measured with Lacoste & Romberg model ‘G’ and ‘D’ land gravimeter and model ‘S’ sea gravimeter in 2001. We selected 2158 gravity measurements near Hong Kong territories together with 640 gravity measurements in Hong Kong for the determination of Hong Kong geoid. Figure 3 shows the distribution of these gravity measurements.

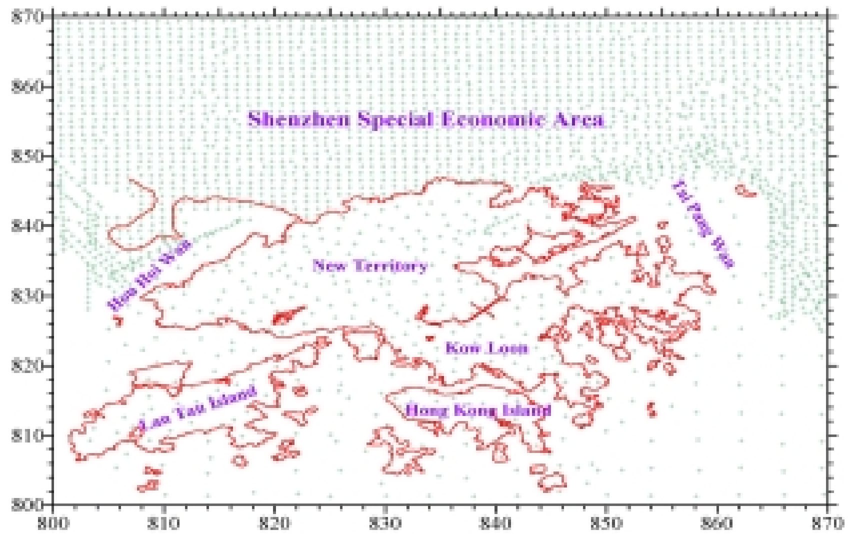

Figure 3 Distribution of gravity observations over Hong Kong and its neighboring region

3. SELECTION OF THE BEST GEO-POTENTIAL MODEL

In the determination of local or regional geoid, the remove-restore approach is usually employed. The medium and long wavelength components of local gravity field is first modeled by a high degree geo-potential model. For this purpose the geo-potential model with the best fitting to the area of interest is preferred. There are many high degree global geo-potential models, including the OSU series (OSU89a, OSU89b and OSU91a), the GFZ series (GFZ93a, GFZ93b and GFZ95a), the NASA GSFC and NIMA joint model EGM96 (Pavlis, 1996), WDM94 developed by Wuhan Technical University of Surveying and Mapping (Ning et al., 1994), GPM98CR (Wenzel, 1998), and etc. We evaluated three models (i.e., EGM96, WDM94, and GPM98CR) using the following data: 43 GPS/Leveling observations and 640 gravity measurements in Hong Kong, and 62 GPS/Leveling observations and 4623 gravity measurement in Shenzhen.

Let h and H denote the GPS-derived ellipsoidal height and the leveled height of a station, respectively. The geoid height at the station can be calculated by

$$N^{obs} = h - H \tag{1}$$

Value N^{obs} so obtained here called observed geoid height. The geoid height N^{GM} from a geo-potential model can be formulated by (Heiskanen and Moritz, 1967)

$$N^{GM} = \frac{GM}{r \cdot \gamma} \sum_{n=2}^{n\max} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\sin\varphi) \tag{2}$$

where GM is the geocentric gravitational constant, γ is the normal gravity of the computation point, a is the long radius of reference ellipsoid, *nmax* is the maximum degree of geopotential model, φ, λ and r are the geocentric latitude, longitude and radial distance of the computation point, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the fully normalized coefficients of the anomalous potential, and $\overline{P}_{nm}(\sin\varphi)$ is the fully normalized associated Legendre functions, given degree n and order m. The differences between the observed geoid heights and those from geo-potential model at the same position can be computed by

$$\Delta N = N^{obs} - N^{GM} \tag{3}$$

The values ΔN reflect the goodness of fitting. Table 1 lists the statistic values of ΔN for three geo-potential models in Hong Kong and Shenzhen. To remove the possible systematic biases, four parameter transformation is applied, i.e.,

$$N^{obs}(x, y) = N^{GM}(x, y) + a_0 + a_1 x + a_2 y + a_3 xy + v \tag{4}$$

where $a_i (i = 0,1,\cdots)$ is unknown parameters, v denotes a residual random noise term, x and y are HK80 grid coordinates or Shenzhen local grid coordinates in kilometer.

It is shown that the STD from geo-potential models EGM96, WDM94 and GPM98CR are ±0.039m, ±0.037m and ±0.030m in Hong Kong, and ±0.052m, ±0.032m and ±0.095m in Shenzhen, respectively. In general the accuracy of geoid heights from WDM94 is better. This is because more observed gravity data covering the territory of China were used to develop WDM94 geo-potential model.

Gravity measurements can also be used to evaluate these geo-potential models. The gravity observations are reduced to the free-air gravity anomalies Δg^{obs} on the geoid (approximately the mean sea level). Furthermore the gravity anomalies Δg^{GM} on the gravity stations from geo-potential model up to degree 360 for EGM96 and WDM94, and 720 for GPM98CR can be calculated by (Heiskanen and Moritz, 1967)

$$\Delta g_{GM} = \frac{GM}{r^2} \sum_{n=2}^{n\max} \left(\frac{a}{r}\right)^n (n-1) \sum_{m=0}^{n} (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\sin\varphi) \tag{5}$$

The difference between the observed gravity anomalies and those computed from geo-potential model can be obtained by

$$dg = \Delta g^{obs} - \Delta g^{GM} \tag{6}$$

As ΔN, values dg can also reflect the goodness of fitting. Table 2 lists the statistic values of dg. From Table 2 we can see the STD values for EGM96, WDM94 and GPM98CR are ±14.136mGal, ±13.559mGal and ±15.130mGal in Hong Kong, respectively, and ±8.913mGal, ±8.613mGal and ±11.750mGal in Shenzhen. These results also indicate that WDM94 is better than others. Therefore WDM94 is a preferred reference geo-potential model for the determination of new Hong Kong geoid.

Table 1 Statistics of values ΔN in Hong Kong (HK) and Shenzhen (SZ) (Unit : meter)

Geopotential model		Max		Min		Mean		STD	
		HK	SZ	HK	SZ	HK	SZ	HK	SZ
EGM96	before	0.356	0.033	-0.119	-0.339	0.100	-0.137	±0.132	±0.080
	after	0.090	0.127	-0.105	-0.117	0.000	0.000	± 0.039	±0.052
WDM94	before	0.580	0.895	0.239	0.125	0.407	0.416	±0.073	±0.140
	after	0.080	0.076	-0.118	-0.077	0.000	0.000	± 0.037	±0.032
GPM98CR	before	0.284	0.308	-0.280	-0.518	0.038	-0.158	±0.160	±0.201
	after	0.082	0.227	-0.064	-0.216	0.000	0.000	± 0.030	±0.095

Note: 'before' denotes the values before systematic bias removed, and 'after' denotes the values after systematic bias removed.

Table 2 Statistics of values dg in Hong Kong and Shenzhen (Unit: mGal)

Geopotential model	Max		Min		Mean		RMS		STD	
	HK	SZ	HK	SZ	HK	SZ	HK	SZ	HK	SZ
EGM96	51.653	64.509	-29.729	-109.491	-10.067	-8.133	17.346	12.065	±14.136	±8.913
WDM94	57.842	68.619	-23.139	-103.433	-5.640	-3.718	14.675	9.380	±13.559	±8.613
GPM98CR	49.287	65.745	-33.995	-108.023	-10.068	-6.231	18.163	13.299	±15.130	±11.750

4. GRAVIMETRIC GEOID DETERMINATION

In theory, local gravimetric geoid can be determined by solving the geodetic boundary value problem (such as Stokes' problem or Molodensky' problem) using gravity anomaly covering the area of interest as boundary value. But in practical computation the remove-restore technique has been widely used to improve the Stokes integral solution, especially in mountain area (see, e.g. Moritz, 1983; Sideris and Forsberg, 1990; Tziavos et al., 1992; Forsberg and Tscherning, 1997; Zhang and Featherstone, 1997). With the technique the long-medium wavelength component derived from a geopotential model and the short wavelength component due to the topographic effect are removed mathematically from the observed gravity anomalies. And then the Stokes′ integration of the remaining parts of the gravity anomalies (called residual gravity anomaly) provides the residual geoid height. The final geoid heights of points are obtained by restoring the previous removed components, i.e., the gravimetric geoid at point $P(\varphi_P, \lambda_P)$, denoted as $N^G(\varphi_P, \lambda_P)$, can be expressed as

$$N^G(\varphi_P, \lambda_P) = N(\varphi_P, \lambda_P) + N^{GM}(\varphi_P, \lambda_P) + N^T(\varphi_P, \lambda_P) \qquad (7)$$

where, $N^T(\varphi_P, \lambda_P)$ is the topographic indirect effect on the geoid due to the second method of Helmert's condensation of the topography (Wicheincharoen, 1982), $N^{GM}(\varphi_P, \lambda_P)$ is computed from a geopotential model by equation (2) and $N(\varphi_P, \lambda_P)$ is the contribution of residual gravity anomaly $\delta g(\varphi,\lambda)$, computed by the Stokes′ integration. For the detail readers are referred to (Yang and Chen, 2001).

The grid terrain correction with spacing 100m was first computed using the mass prism topographic model and 100% zero padding technique (Sideris, 1990; Sideris and Li, 1992; Li, 1993). The grid terrain corrections with grid interval 500 meters and one kilometer were interpolated respectively from the 100 meters grid. These terrain corrections, the gravity anomalies from WDM94 by equation (5) and the observed gravity anomalies were employed to construct the residual gravity anomalies of 1km grid. Figure 4 provides the residual gravity anomalies of 1km grid. Secondly, using original spherical Stokes kernel function and 1-D FFT technique (Haagmans et al., 1993; Li, 1993; Sideris and She, 1995) the residual geoid was computed. Figure 5 show the residual geoid with 1km grid. Finally, the gravimetric geoid is obtained by restoring the previous removed components. Figure 6 displays the gravimetric geoid with resolution of 1km, and the statistic for different geoid components was listed in Table 3.

Table 3 Statistic of different geoid components for 1 km grid (unit : meter)

Item	Max	Min	Mean	STD
Residual geoid	-0.065	-0.275	-0.162	±0.047
WDM94 to degree and order 360	-0.236	-3.893	-2.064	±0.835
Terrain indirect effects on geoid	0.000	-0.024	-0.001	±0.002
Terrain direct effects on geoid	0.070	0.015	0.034	±0.010
Gravimetric geoid	-0.325	-4.058	-2.260	±0.868

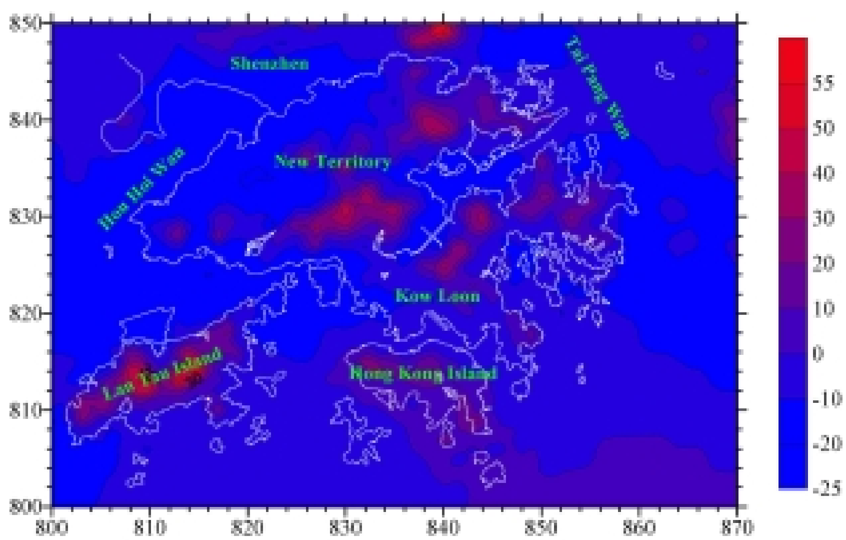

Figure 4 Residual gravity anomalies of Hong Kong at 1km grid
(HK80 Grid Coordinates, Contour interval 10mGal)

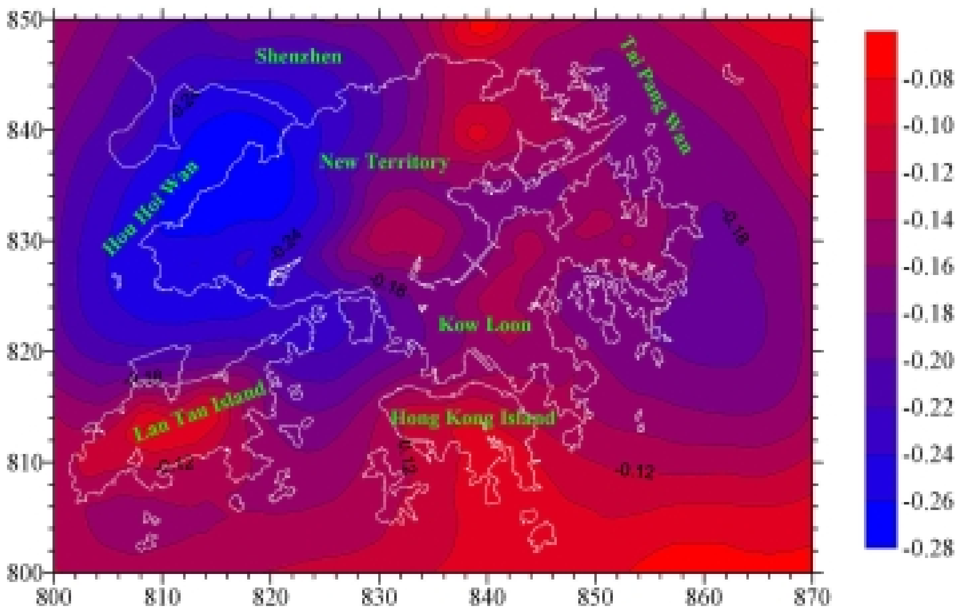

Figure 5 Residual geoid at 1 km grid (HK80 Grid Coordinates, contour interval 0.02m)

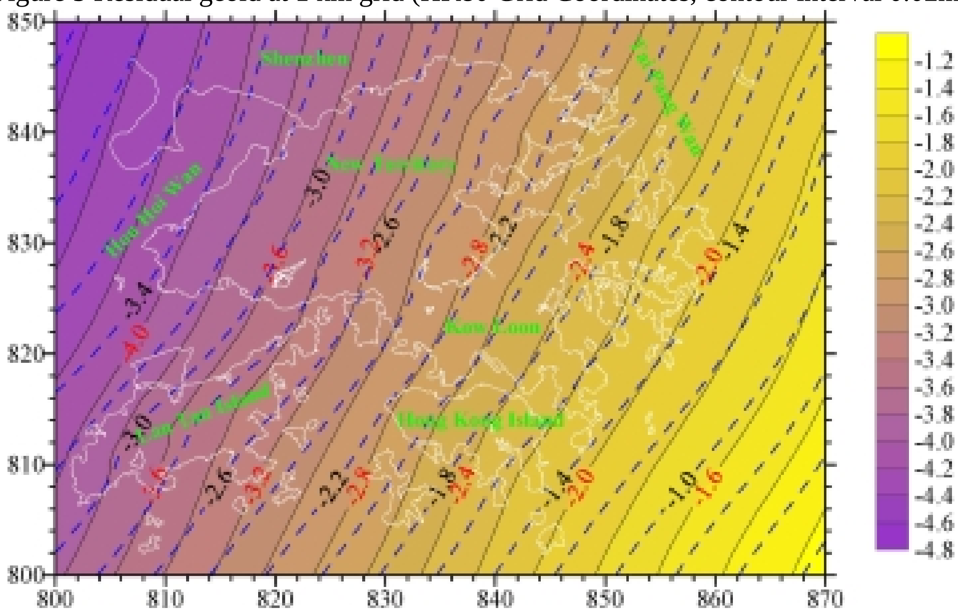

Figure 6 Gravimetric geoid at 1km grid (HK80 Grid Coordinates, contour interval 0.2m)
(Solid-4 parameter model transformed geoid, Dash-original gravimetric geoid)

The accuracy of such derived gravimetric geoid is always estimated by comparing their geoid heights with that derived from GPS/Leveling in relative or absolute sense. The comparison results show that the standard deviation of the gravimetric geoid is ±7.6 cm, and the relative accuracy can arrive at about 4ppm for baseline length longer than 50 km. After the systematic bias removed by equation (4) the standard deviation of transformed geoid is ±1.9cm, and its relative accuracy is better than 1ppm for baseline length longer than 40km (see Figure 7). These results also illustrate that the gravimetric geoid can be improved significantly by precise GPS/leveling data.

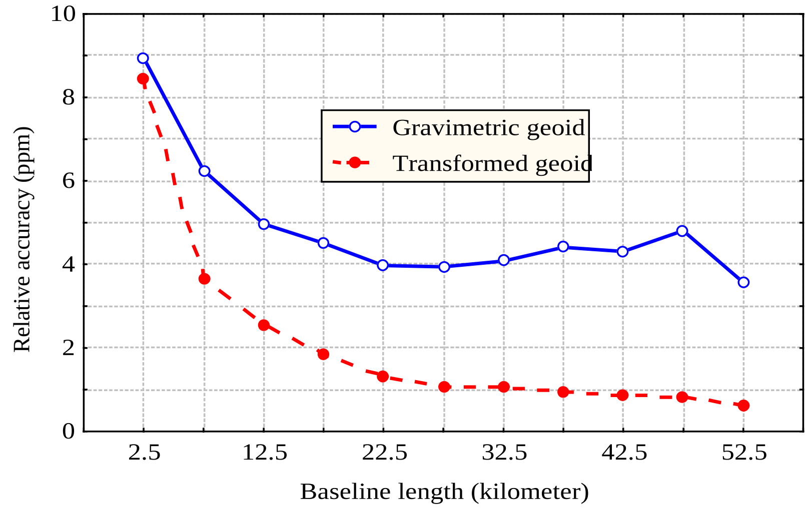

Figure 7 Relative accuracy of gravimetric geoid and 4-parameter transformed geoid

In this study the effects of gravity data outside the computed area was investigated. The Hong Kong geoid was determined both with gravity measurements in Shenzhen and without the data there. The differences are given in Table 4, where the transformed geoid means the systematic biases were removed using the GPS/Leveling data. It can be seen that the differences are significant for some parts of the area. Figures 8 and 9 show these differences. It is clear that the significant differences are in the northern part of the territory.

Table 4 Differences between the gravimetric geoid heights at 1 km grid points with and without Shenzhen gravity data included (unit: meter)

	Max	Min	Mean	STD
Gravimetric geoid	0.070	-0.020	0.007	±0.014
Transformed geoid	0.062	-0.015	0.003	±0.013

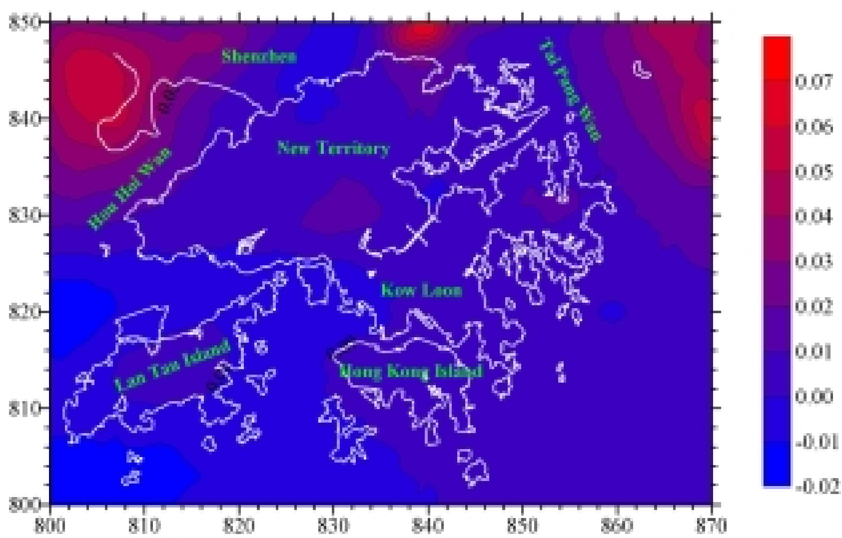

Figure 8 Differences between the gravimetric geoid with and without Shenzhen gravity data included before systematic biases removed (HK80 Grid Coordinates, contour interval 0.01m)

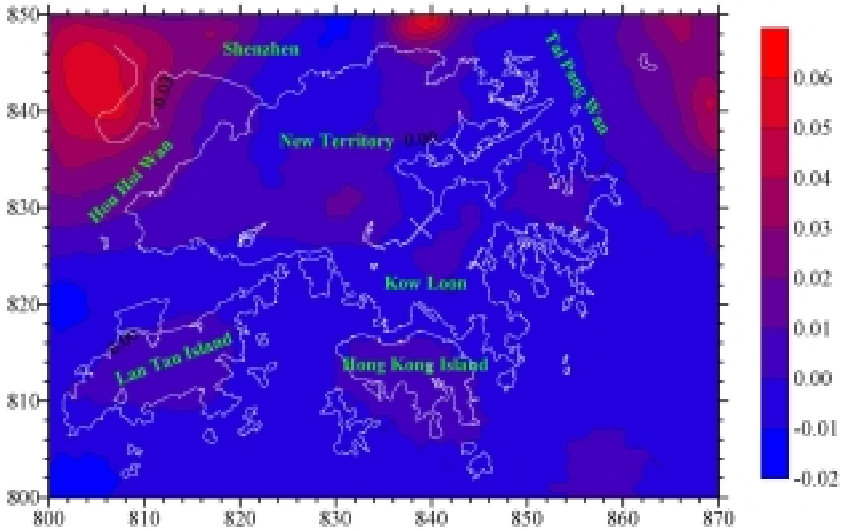

Figure 9 Differences between the gravimetric geoid with and without Shenzhen gravity data included after systematic biases removed (HK80 Grid Coordinates, contour interval 0.01m)

5. THE GEOID DETERMINED WITH HETEREOGENEOUS DATA

To precisely determine local geoid a strategy to take advantage of all the data/information is discussed in detail in (Chen and Yang, 2001). The gravimetric geoid was first constructed using the remove-restore technique with WDM94 as reference geo-potential model, terrain correction applied, and gravity data both in Hong Kong and Shenzhen. The possible systematic biases were then removed with GPS/Leveling data. To improve the accuracy of the geoid, the GPS/Leveling data should be fully utilized. Therefore local Shepard surface interpolation method is used to interpolate the residuals at one kilometer grid, i.e.,

$$\delta N(\varphi_P, \lambda_P) = \begin{cases} \sum_{i=1}^{n} \delta N(\varphi_i, \lambda_i) \cdot K(r_i)^m \Big/ \sum_{i=1}^{n} K(r_i)^m & , \ r_i \neq 0 \\ \delta N(\varphi_i, \lambda_i) & , \ r_i = 0 \end{cases} \tag{8}$$

where δN is the difference between the GPS/leveling derived geoid height and gravimetric geoid height, r_i is the distance between interpolating point (φ_P, λ_P) and known point (φ_i, λ_i), m the fitting power should be integer $(m = 1,2,\cdots)$, and $K(r_i)$ the kernel function or so-called weighting function, i.e.,

$$K(r) = \begin{cases} 1/r & , 0 < \mathrm{r} \leq \mathrm{D}/3 \\ \dfrac{27}{4D}(\dfrac{r}{D} - 1)^2 & , \mathrm{D}/3 < r \leq D \\ 0 & , \mathrm{r} > \mathrm{D} \end{cases} \tag{9}$$

with D interpolating or searching radius. Such derived geoid is called hybrid or combined geoid. Figure 10 is the hybrid geoid with 1km grid. This geoid will be evaluated using independent precise GPS/leveling data in near future.

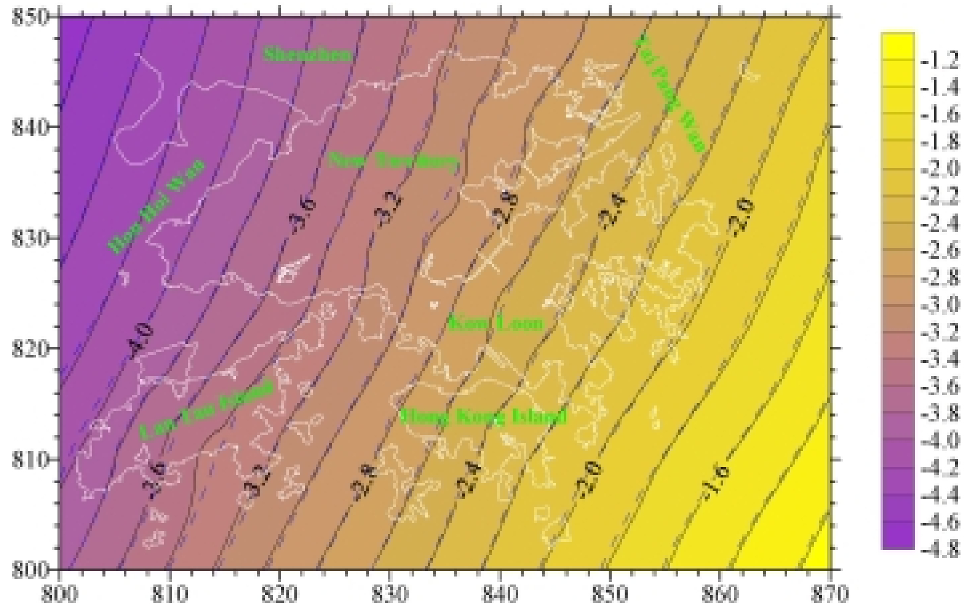

Figure 10 Combined geoid at 1km grid (HK80 Grid Coordinates, contour interval 0.2m)
(Solid-hybrid geoid, Dash- transformed geoid)

6. CONCLUSIONS

This paper summarizes the practical computation of the new Hong Kong geoid. On the basis of remove-compute-restore approach, the gravimetric geoid with resolution of 1km covering the 50km by 70km territory of Hong Kong, has been computed using global spherical harmonic geopotential model WDM94 to degree and order 360, gravity anomalies and digital terrain model covering the whole territory of Hong Kong and its neighboring region. Original Stokes spherical kernel function and 1-D FFT technique were employed to determine the residual geoid. Comparison with the geoid heights derived from GPS/leveling data shows systematic biases exist between the two types of geoid heights. However, removal of the systematic biases using high precise GPS/leveling data can significantly improve the gravimetric geoid. Furthermore, the hybrid geoid was computed by interpolating the residuals at grid points with Shepard surface fitting method. The final geoid will be evaluated using independent precise GPS/leveling data in near future.

ACKNOWLEDGEMENTS

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