Height Determination by GPS – Accuracy with Respect to Different Geoid Models in Sweden

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Key words: geoid, GPS levelling, kriging, height determination.

ABSTRACT

In this paper different gravimetric geoid models have been evaluated: on a regional bases utilising the Swedish GPS reference network SWEPOS; and locally, in a specific research area utilising the results from a GPS campaign conducted in the area.

The research area is situated in central Sweden having a size of approximately 100x300 km. In the area GPS measurements (with average baseline length of 13 km) have been conducted at well established benchmarks with known orthometric heights.

To find out if a geometric geoid model, based on the GPS measurements, would be a better height corrector surface for the research area than the gravimetric geoid models, such a model has been computed with geostatistical methods, i.e. universal kriging. The model has been experimentally evaluated utilising a Swedish GPS campaign called RIX95.

It was found, when the geometric geoid model is used, that the absolute accuracy of GPS levelling is ±14 mm and the relative accuracy ±10 mm (on a 10 km long baseline). It is concluded that the geometric geoid model is the best height corrector surface for the research area among the studied models.

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1. INTRODUCTION

The satellite based global positioning system Navstar-GPS (from now on referred to as GPS only) have had a tremendous impact on geodesy and surveying since its introduction some 20 years ago. The most widespread use of GPS in geodesy has, however, been on obtaining two-dimensional positions, leaving the third dimension (height) out mainly because of the problems associated with different reference systems. Heights obtained by GPS are above an ellipsoid and are fundamentally different from traditionally obtained heights which are given with respect to the geoid.

Mathematically, there is a simple relation between the two reference systems (where we have neglected the deflection of the vertical and the curvature of the plumb line, see Fig. 1.1):

$$H = h - N \tag{1.1}$$

In practice, the expression reflects the possibility of GPS levelling, because it states that if the geoidal height N is known, the orthometric height H (or normal height, depending on the definition of the geoid, but hereafter referred to as orthometric height) can be obtained from ellipsoidal height h determined by GPS. Obtaining orthometric heights this way, could in certain circumstances, depending on the required accuracy, replace conventional spirit levelling and thus make the levelling procedure cheaper and faster.

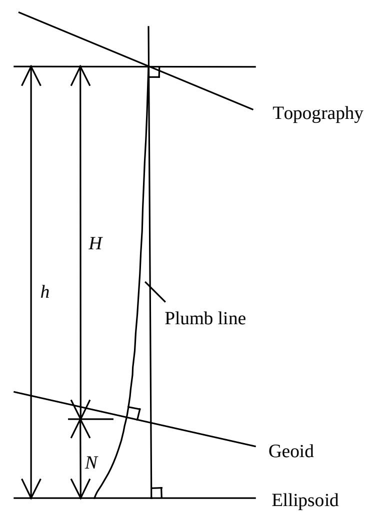

Figure 1.1 The relation $H = h - N$

The crucial part of this method is the geoidal height, which normally is obtained with a lower accuracy than the ellipsoidal height and thus affecting the accuracy of the orthometric height. If the available gravimetric geoid model does not meet the required accuracy, the problem can be overcome by creating a local model of the geoid, a geometric geoid model, by measuring h_i (with GPS equipment) on several points (i) with known orthometric heights H_i (such as the benchmarks constituting the Swedish levelling network). Such a geoid model, created by GPS/levelling (an expression hereafter used when combining ellipsoidal heights and spirit levelling in order to obtain geoidal heights) from the differences $h_i - H_i$, is sometimes referred to as a "GPS-geoid" model (*Forsberg* 1998).
Alternatively, a local gravimetric geoid model can be constrained to points obtained through GPS/levelling and thus combine the high relative accuracy, which normally a local

gravimetric geoid model has, with a good datum reference.

Today it does not seem possible to replace precise levelling (assumed accuracy < 1 mm/km^{½}) by GPS levelling, at least not on short distances, but with refined observation and data analysis techniques it certainly will, where appropriate, in the future.

2. COMPARISON OF DIFFERENT GEOID MODELS

On a regional bases, i.e. all over Sweden, three different geoid models have been used for the comparison of accuracy with respect to GPS/levelling. Locally, in a specific research area, the same models, and additionally a GPS-geoid, have been used. The three geoid models are: EGM96, the most recent global geoid model (*Lemoine et al*. 1997); NKG96, a Nordic geoid model produced within the frame of the Nordic Commision of Geodesy (NKG) (*Forsberg et al.* 1996); and SWEN98L, a Swedish geoid model, or rather, a height correction model, since the Fennoscandian land uplift has been included in the model. SWEN98L has been produced by the National Land Survey of Sweden (NLS).

The residuals, obtained from a GPS/levelling comparison with the different geoid models are taken as an indication of the geoid model accuracy. To find out if the models are affected by datum inconsistencies or systematic effects, we apply a four-parameter model according to Eq. (2.1) to the GPS/levelling points and perform a least squares adjustment to estimate the residuals.

$$\Delta N_i = N_{grav(i)} - N_{GPS(i)} = a_0 + a_1\cos(\varphi_i)\cos(\lambda_i) + a_2\cos(\varphi_i)\sin(\lambda_i) + a_3\sin(\varphi_i) + v_i \quad (2.1)$$

where $N_{grav(i)}$ and $N_{GPS(i)}$ are the geoidal heights at point (*i*) obtained from the gravimetric geoid model respectively from the GPS/levelling. a_0 to a_3 are the four unknown parameters, φ_i and λ_i are the latitude respectively the longitude of the point *i* and v_i is the residual geoid error. Eq. (2.1) correspond to a datum transformation model described in *Heiskanen and Moritz* (1967, Chapters 5 - 9).

2.1 Comparison on a Regional Bases

19 stations in the Swedish GPS reference network SWEPOS (see Fig. 2.1) are used for the regional comparison. The average distance between two nearby stations in the reference network is appr. 130 km and the longest distance between two stations is appr. 1200 km (between no 1 and 19 in Fig. 2.1).

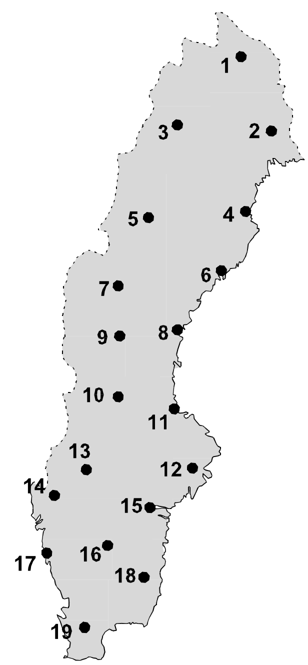

Figure 2.1 The distribution of the 19 SWEPOS stations over Sweden.

The results of the comparison can be seen in Table 2.1 (without fitting by the four-parameter trend function) and in Table 2.2 (after fitting).

Table 2.1 Statistics of differences between different geoid models and GPS/levelling derived geoidal heights on 19 SWEPOS stations. Units in cm.

Geoid Model/ Statistics	EGM96	NKG96	SWEN98L
Min	-9.9	-15.2	-9.9
Max	44.5	12.9	12.1
Mean	+13.1	-3.9	+0.2
SD	±14.4	±6.3	±5.2
RMS	±19.2	±7.3	±5.1

Table 2.2 Statistics of differences between different geoid models and GPS/levelling derived geoidal heights on 19 SWEPOS stations after fitting by a four-parameter trend function. Units in cm.

Geoid Model/ Statistics	EGM96	NKG96	SWEN98L
Min	-13.8	-8.2	-8.3
Max	14.1	8.0	8.4
Mean	0.0	0.0	0.0
RMS	±8.7	±4.5	±4.6

A significant improvement, from a RMS-value of fit of ±19.2 cm to ±8.7 cm, can be seen on the EGM96 geoid model after applying a four-parameter transformation. An improvement which reveals the systematic effects that influence the EGM96 geoid model over Sweden. Effects which hereby also influence the NKG96 geoid model, since it is based on EGM96, as can be seen by comparing the statistics in Table 2.1 and 2.2. The fitting process to 76 GPS/levelling stations, which was conducted on NKG96 in order to arrive at SWEN98L, improve the RMS of fit from ±7.3 cm to ±5.1 cm, while the removal of systematic effects improve NKG96 from a RMS of fit of ±7.3 cm to ±4.5 cm. A fact which makes the NKG96 geoid model the best for Sweden in this comparison. Locally we have reasons to believe that SWEN98L is better because of its constraint to GPS/levelling stations.

2.2 Comparison on a Local Bases

GPS/levelling data for the comparison in the specific research area has been collected through a GPS campaign, conducted by the author (*Mårtensson* 2001), and by orthometric height information obtained from the NLS.

The research area, which can be seen in Fig. 2.2, has been established by forming 8 loops following levelling lines in the Swedish precise levelling network. GPS height measurements have thus been conducted at well established benchmarks, all together 91 benchmarks tied together by 205 baselines.

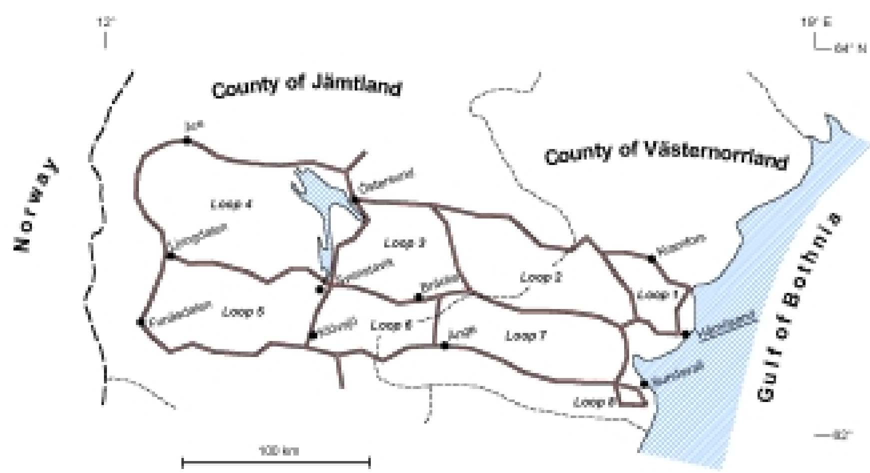

Figure 2.2 The research area in central Sweden.

The results from a least squares adjustment, where the entire GPS-net has been constrained to three nearby situated SWEPOS stations, give in an absolute sense, an RMS error of ±1.3 cm of ellipsoidal heights and in a relative sense, ±0.7 cm on a baseline length of 10 km.
For the relative accuracy, i.e. ellipsoidal height differences, we have used a function based on the general formula used to express accuracy for electronic distance measurements:

$$\sigma^2 = a^2 + b^2L^2 \qquad (2.2)$$

where L is the baseline length and σ the corresponding standard error. a and b are regression coefficients. The regression coefficients for the campaign discussed above are a = 6.7 ±0.2 mm and b = 0.16 ±0.01 ppm.

The geoid models which were used for comparison on a regional bases have now been compared to the GPS/levelling achieved geoidal heights in the research area. The comparisons have been conducted in an absolute, as well as in a relative sense. The results of absolute comparisons can be seen in Tables 2.3 and 2.4.

Table 2.3 Statistics of differences between different geoid models and GPS/levelling derived geoidal heights of the research area. Units in cm.

Geoid Model/ Statistics	EGM96	NKG96	SWEN98L
Min	-40.6	-12.7	-9.9
Max	27.5	6.9	10.0
Mean	+5.7	-5.6	-1.3
SD	±13.7	±3.1	±3.5
RMS	±14.7	±6.4	±3.7

Table 2.4 Statistics of differences between different geoid models and GPS/levelling derived geoidal heights of the research area after fitting by a four-parameter trend function. Units in cm.

Geoid Model/ Statistics	EGM96	NKG96	SWEN98L
Min	-40.3	-6.6	-6.7
Max	22.3	11.5	11.7
Mean	0.0	0.0	0.0
RMS	±12.8	±2.6	±2.6

The EGM96 geoid model is not as obviously improved in the research area, after fitting, as was the case when compared to the SWEPOS network in Section 2.1 (cf. Tables 2.1 and 2.2). NKG96 is, compared to SWEN98L, suffering from a bias in the research area, we improve the RMS of fit from ±6.4 cm to ±3.0 cm by removing the bias only. For SWEN98L the corresponding improvement is from ±3.7 cm to ±3.5 cm. The improvements are further extended to ±2.6 cm, for both models, when fitted by the four-parameter transformation. Both models are thus in equivalence for the research area, provided systematic effects are removed. When not fitted, SWEN98L is the best geoid model for the research area with a RMS of fit of ±3.7 cm. The resemblance between NKG96 and SWEN98L is revealed in Table 2.4, verifying that the origin of SWEN98L is NKG96.

2.2.1 The RIX95 GPS-Project

Parallell to the GPS campaign conducted in the research area, a similar campaign called RIX95, was conducted by the NLS in neighbouring areas. One part of NLS's campaign cover approximately 60% of the research area (eastern part of Fig. 2.2) and as such suitable for comparison and validation of results obtained in the research area. The GPS campaign conducted by the NLS is far more comprehensive than the campaign conducted by the author in the research area.

Comparing the part of the whole RIX95 campaign mentioned above with the research area gives that there are 662 GPS-observed points, of which 133 are connections to the Swedish height system (RH70), tied together by 3923 GPS baselines in the RIX95 area. In the research area there are 91 GPS-observed points, all connected to RH70, tied together by 205 baselines.

An evaluation of the specific part of the RIX95 campaign which covers the research area gives the following results: RMS of ellipsoidal heights ±0.9 cm (compared to ±1.3 cm for the research area); and regression coefficients of ellipsoidal height differences (cf. Eq. 2.2) a = 6.06 ±0.03 mm, b = 0.300 ±0.003 ppm (compared to a = 6.7 mm, b = 0.16 ppm for the research area). Ellipsoidal heights on the only point which are in common in both campaigns, show a difference of not more than 4 mm.

Table 2.5 shows the statsistics for the RIX95 campaign part which covers the research area.

Table 2.5 Statistics of differences between different geoid models and GPS/levelling derived geoidal heights of the RIX95 area. Units in cm.

Geoid Model/ Statistics	EGM96	NKG96	SWEN98L
Min	-9.4	-12.5	-9.9
Max	34.4	2.0	5.3
Mean	9.6	-6.4	-3.3
RMS	±13.0	±7.0	±4.5

The results in Table 2.5 confirms the results obtained in the research area (cf. Table 2.3), with somewhat less accurate statistics for SWEN98L. A comparatively gross bias can be seen for SWEN98L in the RIX95 area, if it is removed, the RMS of fit become ±3.0 cm.
The relative accuracy of the geoid models EGM96, NKG96 and SWEN98L can be estimated utilising the RIX95 campaign. The reason for utilising RIX95 is the possibility to later on make a comparison to the evaluation of the geometric geoid model HiG00 (cf. Section 2.3.1).
26 baselines which are suitably situated inside, or close to, the research area have been chosen from the RIX95 campaign, the average length of the baselines is 8540 m, differing from 1974 m to 21006 m. The statistics in Table 2.6 are based on a comparison of calculated orthometric height differences with height differences obtained from official orthometric heights. Where appropriate, the official orthometric height differences have been corrected for the relative land uplift between the baseline endpoints.

Table 2.6 Statistics of relative accuracy of different geoid models. a and b are in accordance with Eq. (2.2).

Geoid model/ Statistics	EGM96	NKG96	SWEN98L
a (mm)	20.3±11.5	8.65±2.5	9.26±2.3
b (ppm)	3.5±0.5	0.79±0.2	0.66±0.2
SD on L=10 km (cm)	±4.0	±1.2	±1.1

Here we can see the regional improvement of EGM96, being a long-wavelength geoid model,

by adding short-wavelength characteristics, a result of which is the NKG96 geoid model.
The resemblance between NKG96 and SWEN98L is again revealed, this time in the relative accuracy of the two geoid models, ±1.2 cm respectively ±1.1 cm for a 10 km long baseline.

2.3 GPS-geoid Modelling of the Research Area

Since the research area is not completely covered with observations, we have to create a continuous surface from point data by some interpolation (or prediction) method, the choice of which is crucial for the result. Some studies from recent years emphasising this fact can be mentioned, like *Jiang and Duquenne* (1996), *Zhong* (1997), *Kotsakis and Sideris* (1999) and *Lee and Mezera* (2000). A comprehensive compilation of different methods can be found in *Watson* (1992) and *Burrough and McDonnell* (1998).

For the research area, we are going to apply what is called local interpolation (*Burrough and McDonnell* 1998, p. 103) by turning to geostatistics to choose kriging.

Why do we choose kriging instead of the well known least squares collocation, the latter favoured by geodesists?
The answer lies in the availability of software, as kriging has become an extremely important interpolation tool in geographical information systems (GIS) and, as such, has been given a lot of attention from scientists and software producers. A thorough textbook on kriging in general is *Stein* (1999), on applied geostatistics is *Isaaks and Srivastava* (1989) and on kriging in GIS in particular is *Burrough and McDonnell* (1998). A comparison of kriging and collocation has been made by *Dermanis* (1984). Software like ArcInfo (ESRI Inc.) and Surfer (Golden Software, Inc.), for instance, have excellent modules dealing with kriging.

Both kriging and least squares collocation are generalised estimation methods combining the behaviour of a systematic part and two random parts. A vocabulary difference between kriging and least squares collocation lies in the treatment of the covariance structure of the random field, where in kriging semivariograms are used (*Blais* 1982, p. 327) to estimate weights and in least squares collocation covariance functions (*Moritz* 1973, p. 26).

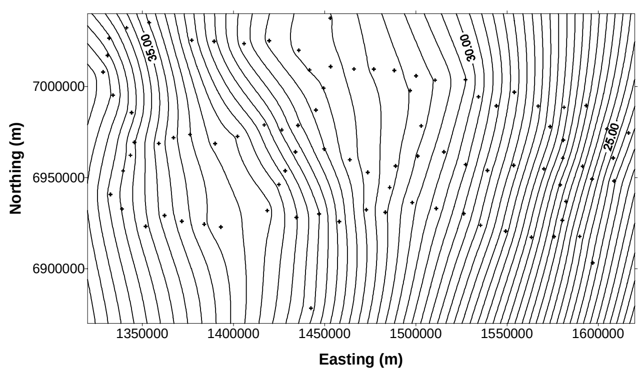

Figure 2.4 The geoid model HiG00 of the research area. Isoline units in m with an equidistance of 0.25 m, crosses marks GPS-observed points.

The basis for kriging as an optimal interpolator are contained in regionalised variable theory, where it is assumed that the spatial variation in the phenomenon represented by “height” is statistically homogeneous throughout the surface. The surface is expressed as the sum of three major components; a structural component, having a constant mean or trend, a random, but spatially correlated component and a spatially uncorrelated residual error term.

We have used a quadratic trend and a linear semivariogram approach, which is considered to be the best for the research area, in a particular method called *universal kriging* (*Mårtensson* 2001, pp. 68-72). The geoid model produced has been designated the name HiG00 (Fig. 2.4).

2.3.1 Error estimation and validation of HiG00

The anticipated accuracy of orthometric height determination from GPS levelling can be obtained by error propagation applied on Eq. (1.1):

$$\sigma_H^2 = \sigma_h^2 + \sigma_N^2 \tag{2.3}$$

In practice, however, we use relative, or differential, GPS observations that provides ellipsoidal height differences with respect to a fixed station with known orthometric height. The change in orthometric height over the GPS baseline *AB* is determined by using a corresponding change in ellipsoid and geoid separation:

$$H_A - H_B = h_A - h_B - (N_A - N_B) \tag{2.4}$$

$$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \tag{2.5}$$

The relative accuracy, which is depending on the baseline length *AB*, can now be obtained by error propagation applied on Eq. (2.5):

$$\sigma_{\Delta H}^2 = \sigma_{\Delta h}^2 + \sigma_{\Delta N}^2 \tag{2.6}$$

where we have omitted the indices *AB*.

For the evaluation of the relative accuracy, we restrict the baseline lengths in this study to be within what we believe is practical use among field surveyors, namely 20 km, thus avoiding long-wavelength errors from the geoid (or the geoid model) in Eq. (2.6), but likely to be present in Eq. (2.3).

The absolute accuracy for different geoid models in the research area are presented in Table 2.3. For the geoid model HiG00 the absolute accuracy can be calculated based on the standard errors stemming from GPS measured ellipsoidal heights (h_M) (i.e. ±1.3 cm, cf. Section 2.2), estimated land uplift values (*lu*) (i.e. ±0.9 cm, cf. *Ekman* (1998)) and known orthometric heights (H_M) (i.e. ±0.6 cm, cf. *Egeltoft* (1996)). HiG00 is also affected by interpolation errors (e_M). Index *M* has been added to indicate that the values originate from the modelling process. The absolute accuracy can then be calculated from:

$$\sigma_{HiG00}^2 = \sigma_N^2 = \sigma_{hM}^2 + \sigma_{lu}^2 + \sigma_{HM}^2 + e_M^2 \tag{2.7}$$

Omitting the degradation caused by interpolation, the absolute accuracy of HiG00 is ±1.7 cm according to Eq. (2.7).
The relative accuracy of HiG00 can be estimated from:

$$\sigma^2_{\Delta HiG00} = \sigma^2_{\Delta N} = \sigma^2_{\Delta h} + \sigma^2_{\Delta H} + (de)^2_M \tag{2.8}$$

Where $(de)_M$ is the differential interpolation error.
By combining the regression coefficients associated with relative accuracy for ellipsoidal height differences according to Section 2.2, i.e. a = 6.7 mm and b = 0.16 ppm, with the accuracy of orthometric height differences, i.e. ±1.63 mm/$\sqrt{km}$ (*Ussiso* 1977), and present the result according to Eq. (2.2), we arrive at a = 7.39 mm and b = 0.39 ppm for HiG00.

σ_h in Eq. (2.3) and $\sigma_{\Delta h}$ in Eq. (2.6) are open, depending on how the ellipsoidal heights, or height differences, have been collected. We have, however, the possibility to make a full analysis and validate HiG00 if we utilise the measurements made by the RIX95 campaign.

For 64 points, belonging to the RIX95 campaign and situated inside the research area, orthometric heights have been calculated by the formula:

$$H_{GPS} = h - N_{HiG00} \tag{2.9}$$

The so obtained orthometric heights H_{GPS} have been compared to the official orthometric heights H_{Off} on the RIX95 points and statistics of the differences are given in Table 2.7 under the column "All covering" (cf. Fig. 2.5).

Table 2.7 Validation statistics for HiG comparing calculated and official orthometric heights. Units in cm.

	All covering	Along lines
Statistics	*dH*	*dH*
Min	-5.4	-4.1
Max	7.1	3.4
Mean	0.2	0.1
RMS	±2.6	±1.4

The anticipated accuracy can be calculated, since:

$$dH = H_{GPS} - H_{Off} = h - N_{HiG00} - H_{Off} \tag{2.10}$$

it follows, that:

$$\sigma^2_{dH} = \sigma^2_{HGPS} + \sigma^2_{HOff} = \sigma^2_h + \sigma^2_{lu} + \sigma^2_{HiG00} + \sigma^2_{HOff} \tag{2.11}$$

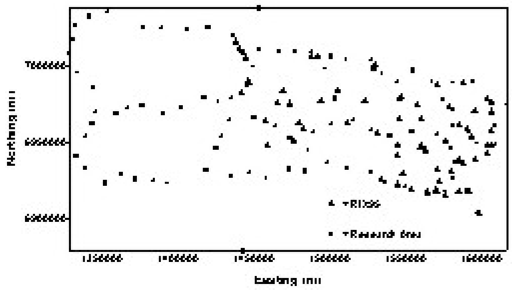

Figure 2.5 RIX95 points “All covering” used to validate HiG00.

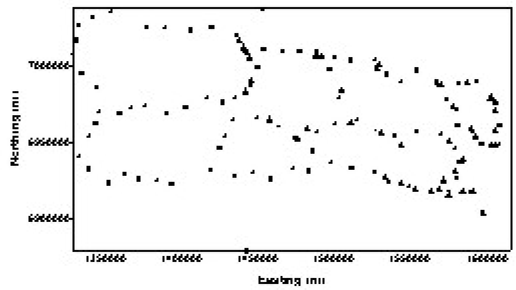

Figure 2.6 RIX95 points ”Along lines” used to validate HiG00.

There is of course a risk that σ_h and σ_{HiG00} are correlated in the above equation, because they both originate from an adjustment where common points, for the research area as well as for the RIX95 area, are held fixed. On the other hand, they do originate from completely different campaigns, with completely different prerequisites, thus we regard σ_h and σ_{HiG00} as being uncorrelated in the forward evaluation process.

With the RMS error from the constrained adjustment of the GPS levelling being ±0.9 cm (cf. Section 2.2.1), the standard error of HiG00 being ±1.7 cm according to Eq. (2.7), the standard error of the land uplift being ±0.9 cm and the standard error of the official height being ±0.6 cm, the last two in accordance with the discussion concerning the accuracy of HiG00, we arrive at $\sigma_{dH} = \pm 2.2$ cm.

Since the RMS value in Table 2.7 (±2.6 cm) is greater than the anticipated value of ±2.2 cm, it probably reveals the interpolation error e_M in Eq. (2.7). It is most likely an effect of the lack of measurements inside the loops in the research area. To find out, all points from the RIX95 campaign which are not very close to the measured lines (within a “corridor” of 5 km) are removed (cf. Fig. 2.6), the statistics of the remaining 42 points is given in Table 2.7 under the column “Along lines”.

The resulting RMS value (±1.4 cm) is small compared to the expected (±2.2 cm), something which might indicate too pessimistic standard errors in the calculation of σ_{dH} in Eq. (2.11). A standard F-test reveals that the variances are not the same at the 95% confidence level, suggesting that there is either something wrong in the error model in Eq. (2.11), or in the official heights of the RIX95 points, or in both. We have reasons to believe in the first assumption.

The RMS value in the “Along lines” column in Table 2.7 does not validate our error model, whilst the RMS in the “All covering” column does, if we consider the difference as being an interpolation error. This holds true, especially when interpolation is required in areas where data points for the model are very sparse (i.e. inside the loops). If interpolation is conducted in areas along “corridors” following the GPS measured lines, the accuracy has experimentally shown to be as good as ±1.4 cm. A result taken as an indication of the attainable accuracy, if the geoid model had been created by a dense network of points, as is the case along the lines.

When HiG00 is the geoid model, the relative accuracy of orthometric height differences obtained from GPS baseline measurement, can be validated utilising the same baselines as for the geoid models examined in Section 2.2.1, i.e. baselines from the RIX95 campaign.

Combining a = 6.06 mm and b = 0.30 ppm for the RIX95 area and a = 7.39 mm and b = 0.39 ppm for HiG00, we obtain for $\sigma^2_{\Delta H}$:

$$\sigma^2_{\Delta H} = 9.56^2 + 0.49^2 L^2 \text{ mm} \tag{2.12}$$

The statistics for a validation of Eq. (2.12) is given in Table 2.8 under the column "All covering".
The anticipated accuracy for a baseline length of 10 km is ±1.1 cm according to Eq. (2.12), to be compared to the experimentally obtained value ±3.5 cm. Again we experience a difference which might reveal errors stemming from the interpolation. Four baselines deviates more than the rest and by removing them we almost halve the SD, but still greater than the anticipated.

To examine if we have the same correlation between "along lines" and relative accuracy as with absolute accuracy, we remove 11 of the 26 baselines which are not along, or nearly along, the measured lines. The result is in the column "Along lines" in Table 2.8.

Table 2.8 Validation statistics of relative accuracy of HiG00. a and b are in accordance with Eq. (2.2).

Statistics	All covering	Along lines
a (mm)	30.1±9.8	8.00±2.7
b (ppm)	1.7±1.2	0.66±0.2
SD on L=10 km (cm)	±3.5	±1.0

Here we managed to validate our error model [Eq. (2.12)], in "All covering" by assigning the surplus of the SD to interpolation errors due to lack of measurements inside the loops, and in "Along lines" almost exact.
The conclusion is, that if the geoid model had been created by a dense network, as is the case along the lines, the relative accuracy of orthometric height differences with GPS levelling is $\pm\sqrt{8^2 + 0.66^2 L^2}$ mm over baseline lengths (L, in km) shorter than 20 km.

3. CONCLUSIONS AND RECOMMENDATIONS

Since we did experience a deterioration of results when applying GPS levelling inside loops in the research area compared to along loops, we recommend the use of a network that resembles a triangulation network in future GPS campaigns where the aim is to obtain surface cover for geometric geoid modelling.

Areas with sparse gravimetric observations, which would benefit from having a geoid model that is better than the best available global model, could be covered by loops resembling a triangulation chain. This is particularly valid for developing countries where there normally exists spirit levelling loops covering great areas but where gravimetric observations are not available.

The height corrector surface which we derived for the research area was done so using geostatistics. In our case we used universal kriging, an interpolation method easily accessible through several software packages. The result of the process utilising universal kriging provided us with a height corrector surface which is better than any presently available gravimetric geoid model. Hereby not discarding the use of gravimetric geoid models, GPS/levelling should be seen, rather as a complement to gravimetric geoid models, than as a competitor.

For the time being, GPS levelling supplements, rather than replaces, precise spirit levelling, but for the future we can anticipate an improvement in GPS height measurements to millimetre level, thus significantly improving the possibility of very accurate orthometric height determination by GPS.

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