Delimitation of Land and Maritime Boundaries: Geodetic and Geometric Bases

Gérard COSQUER and Jean-François HANGOUËT, France

Key words: land boundary demarcation, maritime boundary delimitation, median line, territorial waters, exclusive economic zones, GPS static, rapid static, real time kinematics, Voronoï diagrams.

SUMMARY

State-of-the-art technology is beneficial to the construction of geographic lines and borders, both on the ground on land, and on the map at sea. The paper first explores the usefulness of real time kinematics GPS technique for the setting up of primary and particularly intermediate markers for land borders where the positions have to fulfill two conditions, viz. being lined up and inter-visible, a process which was experimented by IGN (French Institut Géographique National) in the demarcation of the Saudi Arabia – Qatar border in 1997-1998. The second section addresses what Computational Geometry calls Voronoï diagrams, and shows that such geometrical structures, computed on coastlines, provide an elegant computation tool for the delineation of geometric median lines and of maritime zone boundaries.

RÉSUMÉ

La technologie de pointe est particulièrement utile à la construction des lignes et frontières géographiques, qu'elles doivent être abornées sur le terrain, cas des frontières terrestres, ou portées sur une carte, cas des frontières maritimes. Cette communication s'attache d'abord à décrire l'utilité des techniques GPS cinématique temps réel pour la mise en place des points frontière : les points principaux mais aussi, plus notamment, les points intermédiaires qui doivent satisfaire simultanément à deux conditions, à savoir être dans l'alignement de deux points principaux et être inter visibles. Cette technique a été expérimentée par l'IGN (Institut Géographique National, France) à l'occasion de la démarcation de la frontière Arabie Saoudite-Qatar en 1997-1998. Dans sa seconde partie, la communication montre que les structures que la Géométrie Algorithmique nomme *diagrammes de Voronoï*, lorsqu'on les calcule sur les lignes des côtes, permettent de déduire élégamment lignes d'équidistance entre pays et limites de zones maritimes.

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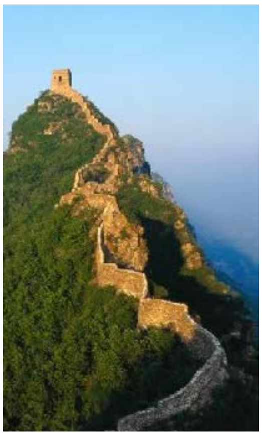
1. INTRODUCTION



From earliest times, civilisations had enclosed their cities to prevent the incursions of invaders. Tenochtitlan, the ancient Mexico was encircled with cactus hedges. In China, the concept of defensive walls extended to encircle entire territories. Started during the Zhou dynasty (1134 BC to 250 BC), the Great Wall was built to stop the barbarians from the north. Between 221 BC to 204 BC, the first emperor of the Qin dynasty ordered to join the previous walls in a gigantic rampart to protect China and to delineate its nothern frontier. This work involved hundreds of thousands people, lasted for centuries and the border extended over 6,700 km.
Nowadays, border demarcation is not only achieved to protect from outsiders but much more to regulate exchanges between neighbouring countries, even if we still have examples of impassable borders as between the two Korea.

1.1 Natural and Artificial Borders

In reference to the "Le Petit Robert" dictionary, the frontier is the limit of a territory which determines its area, eg. the limit separating two states. This line may be natural or artificial. Natural borders (or segments of border) are composed by hydrographic or orographic features while artificial ones always are, according to "Le Petit Robert", conventional ideal lines arbitrarily drawn and marked by conventional signs (monuments, pillars, posts, buoys….).
Natural frontiers can be watersheds, hilltops… which are stable and where monuments can be built or hydrographic lines (rivers, wadi…) which are submitted to potential changes and where monuments cannot be built. Some borders are composite of natural and artificial lines whose segments can be defined by particular and characteristic points of the relief (hillock, table land, confluence…).

Four types of border lines are distinguished on their nature:
1. Hydrographic features (river, lake…)
2. Orographic features (watershed…)
3. Geometric features (meridian, parallel, segment, equidistant line…)
4. Other features (ethnic limit, railway…)

Nature of the border line (in %), from Foucher 1994:119

	1- hydrography	2-orography	3- geometry	4- other
AMERICA	**45**	27	23	5
AFRICA	34	13	**42**	11
ASIA	23	**35**	8	**34**
EUROPE	25	21	5	**50**
WORLD	32	24	23	21

An artificial border is an ideal line which reaches all the perfection (accuracy, in our case) that we can imagine or wish. The border demarcation needs very accurate processes for points determination. This requirement for accuracy is always expressed by "border committees" during delimitation works.

1.2 Border Line Description

The materialization of a land boundary or the determination of a maritime border always take place after an agreement between two (or more) neighboring countries, with the possible participation of other countries, and sometimes after arbitration of the International Court. The agreement stipulates the location of the main border points which can be natural points (eg. highest point of Qarn Abu Wail on the Saudi-Qatar border), defined by coordinates, by some temporary mark, by a description or pointed on a map. The agreement defines also the border line between two main border points which can be a natural line, or a straight artificial line.

The agreement terms being frequently unclear, the way is open to several interpretations by the parties: one difficulty is to fulfill the requirements of strictness and precision.

A maritime border traditionally begins at a point located on the low tide line and extends up to 200 nautical miles (nm), sometimes up to the limit of the continental shelf inside 350 nm.

The maritime border starting point is usually defined as a point located on the low tide line but the last land boundary point is, most of the time, built on the coast slope over the high tide line. The location of the starting point constitutes the first issue to solve in maritime border determination. To overcome this difficulty Saudi-Arabia and Qatar decided to build the land border's first monument as close as possible from the low tide line. A concrete platform surrounded by rock blocks needed to be built in order to raise the monument above sea level, and to protect it from the waves. It is one way to solve the problem, yet neither the cheapest nor the most efficient. Usually the first point of a maritime border is defined only by coordinates.

2. LAND BORDERS

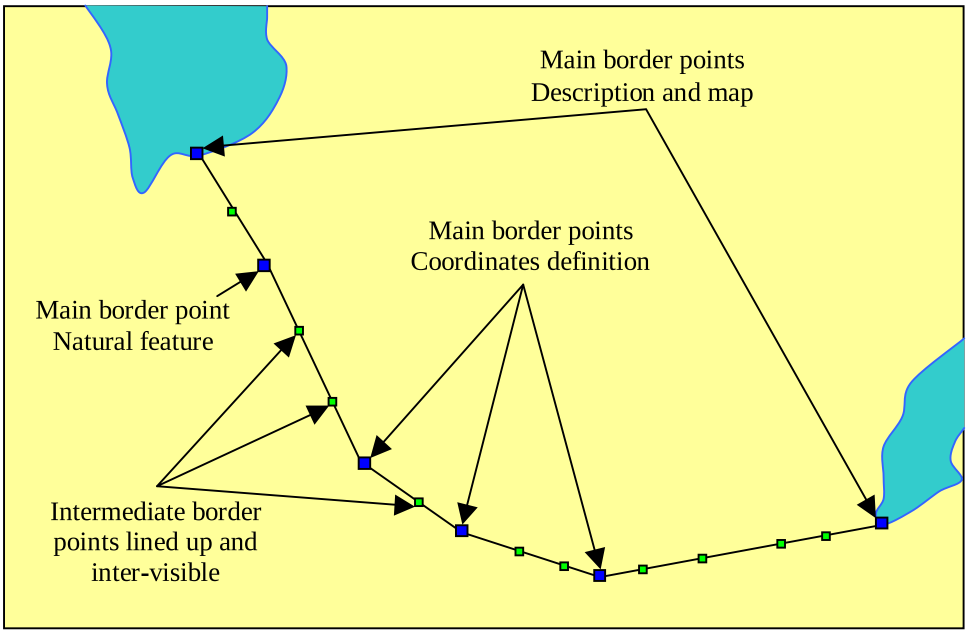
2.1 Datum and Reference Network

In order to guaranty impartiality and sometimes accuracy when any of the countries has no currently referenced geodetic network, a new datum reference network independent from the two countries and based on nearest IGS stations has to be determined in the vicinity of the border main points mentioned in the agreement.

2.2 Main Border Point Set up and Determination

The border coordinates system being established, the main border points defined by their coordinates in the agreement are set up using the "GPS real-time kinematics" observation technique completed with static or rapid static observations. The points not defined by coordinates (natural feature locations, described locations…) are determined by GPS static observations. At this stage, the border points positions are materialized by temporary marks. This operation will be renewed at the time of survey mark fixing at the end of built up.

Schema of Saudi Arabia and Qatar border



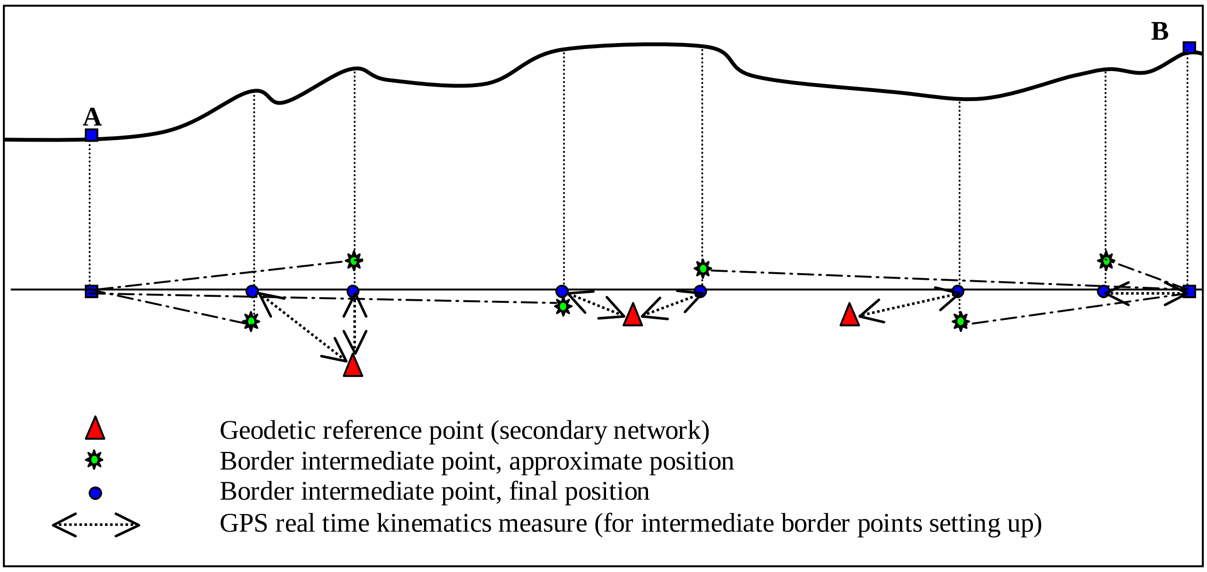
2.3 Intermediate Border Points Set up and Determination

When a border section is a straight segment, it is often planned to set up intermediate points lined up and inter-visible. The stake is to find the location satisfying to the two following criteria:

- being on the geodetic line joining two main border points
- being visible from the two contiguous points.

The GPS real time kinematics technique associated to a geodetic line computation software can be efficient to solve this kind of problematic. The process stages are:

- Increase the density of reference network by setting up a sufficient number of new points nearby the border line to fulfill the conditions of efficient and accurate GPS real time kinematics working (10 to 15 km for the GPS itself and less for wireless transmission, according to transmitter type and terrain topography). The points may be spaced out about 5 km and determined by GPS static technique.
- Make a reconnaissance of approximate positions (5 to 15 m) of the border points satisfying the inter-visibility criteria by using a single GPS.
- Compute the final coordinates of the border intermediate points from the approximate coordinates projected on the geodetic line using a geodetic lines computation software (direct and reverse solution).
- Set up the border points final positions using GPS real time kinematics based on the nearest reference point.
- Control the inter-visibility between the final positions; if ok, make some GPS static or rapid static observations as quality check of the setting up, if not, the process has to be repeated.
- This setting up operation needs to be repeated, as with the main border points, to place the survey mark on the monument top.



3. SEA BOUNDARIES, ON MAPS

3.1 General Points

The delineation of maritime boundaries, just as the delineation of land borders, is a matter of negotiations between states and/or with international authorities. Land, again, if without the land mass, provides the landmarks to prop up the boundaries: most usually, its coast, or a surrogate line known as “baseline”.

The *baseline*, in maritime geography, is defined for its legal purpose as “the line from which the outer limits of a State’s territorial sea and certain other outer limits of coastal State jurisdiction are measured” (Doals, 2000:147). Because sea boundaries are constructed from baselines, these of course are of fundamental importance. Yet any ideal delineation, any mechanical construction on the sole base of the baselines, will inevitably be altered by a wide range of geographical, legal, economic and practical factors:

- geographical factors: *presence* of bays, mouths, reefs, islands, low-tide rocks, coastal indentation, infrastructures…; shape of continental shelf; nature of seabed, subsoil…
- legal factors: concept of proportionality of coast length, of maritime surface area, presence of third state…
- economic factors: hydrocarbon resources, fishery resources, subsoil resources…
- practical factors: navigational access to sea, political and security context, environmental issues…

All these considerations are explained and discussed in detail in the Handbook on the Delimitation of Maritime Boundaries (Doals, 2000).

In this paper, the *baseline* is seen as the cartographic reference coastline as accepted from the State(s) and from which seaward distances will be measured. Only ideal constructs on the baselines are described, focusing on the elegant resolution of purely geometrical problems thanks to Voronoï diagrams: computation of median lines; of maritime zones (territorial sea, contiguous zone, exclusive economic zone, continental shelf); relation with cartographic waterlining; computation of “straight” baselines. Other issues however cannot but benefit from the automated handling, rendering and eliciting of geometrical entanglements: Ridden of the geometrical headaches, and equipped with a tool that makes simulations easy, negotiators can concentrate their efforts on solving other critical issues.

3.2 Maritime Borders

3.2.1 Median Line as Maritime Border

When a borderline between two states has to be found in sea (or, indeed, in lake or in river), a common practice consists of the delineation of the median line, which, given the baselines of the two states, is officially defined as follows: “In the absence of agreement, and unless another boundary line is justified by special circumstances, the boundary is the median line, every point of which is equidistant from the nearest points of the baselines from which the breadth of the territorial sea of each State is measured” (1958 Geneva Convention, quoted in Doals, 2000:14).

3.2.2 Voronoï Diagrams for Maritime Borders

Voronoï diagrams are geometrical structures constructed from a set of elements given in a same metric space (usually, and in this paper, L_2-norm is used). When the elements of the given set verify some “finite” properties (eg. when they are separate points, or whole segments etc.), it happens that Voronoï diagrams can be defined as points minimally equidistant from the elements (Okabe et al. 2000, Bertolotto-Leidinger & Hangouët 2003:88, see illustration with Figure 3.1). This is synonymous indeed with the definition of the median line quoted above, “every point of which is equidistant from the nearest points of the baselines”. In other words, once the geometrical elements of the baselines are given, their Voronoï diagram coincides with the median line.

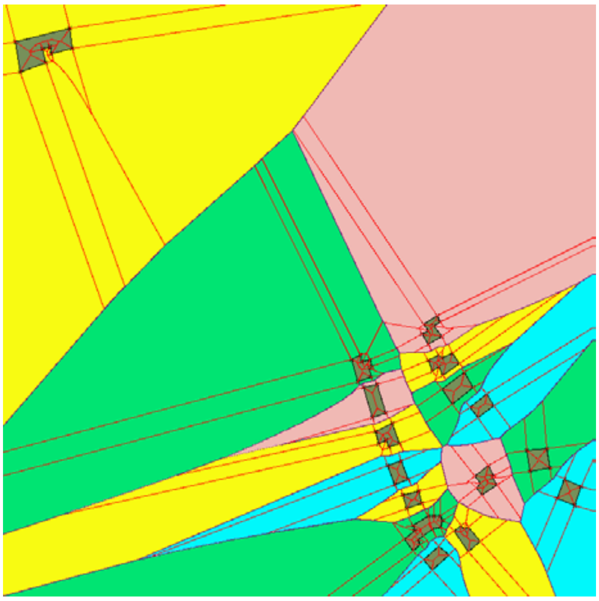


Figure 3.1 Space tessellated by the Voronoï diagram on an “archipelago” of 16 houses.

In any place of any colored cell, you are closer to the building that originates the cell than to any other (the diagram is constructed so as to account also for which corner or wall you are closer to, hence the lines radiating from the buildings).

Two “Voronoï gardens” thus meeting means that you are both equidistant to two buildings, and *closer* to either than to any of the others – ie. that you are on the median lines. Suppose these are buildings on piles in a lake (or square-cornered islands!), the south-eastern group belonging to one state, the north-western lone flyer to another state: the oblique line in-between is the geometric median line.

How to program the computation of Voronoï diagrams, a much-studied issue in the field of Computational Geometry, is not addressed here. It will suffice to say that the algorithmic complexity of the construction, not of the repulsive kind, is $O(n \,.\, \log n)$, when n elements are involved (Okabe *et al.* 2000, Boissonnat & Yvinec 1995). Efficient programs have been designed and described, notably by Fortune 1987 on points, by Held 2000 on points and segments. A precious web resource, for Voronoï theory, applications and tuning on current events is Christopher Gold’s www.voronoi.com web site.
This diagram, being computed on plane point and segment data (ie. on the polylines as stored in habitual GISs, which are used to represent, among others, coastlines and baselines), is geometrically composed of pieces of mediatrix lines (point–point equidistance in the plane), of bisector lines (segment–segment equidistance), and of arcs of parabolas (point–segment equidistance). What geometric and semantic attributes should be stored to describe such outlandish edges and the basic methods (eg. selection of points on the diagram at a given distance from the original data) to make them tractable in habitual GISs is documented in Hangouët 2000.

When the distortions due to the map projection are too great, it may be necessary to compute the equidistance on the sphere: there, the Voronoï diagram is composed of arcs of great circles (point–point equidistance on the sphere, and equidistance between coastal great circle segments) and of arcs of higher-order curves (equidistance between point and coastal great circle segment).

The same diagram can be computed when the elements are not separate (the case, indeed, of the building walls and corners in Figure 3.1 above), ie. when national coasts are "adjacent", in the geographical terminology of maritime borders.
In this case, it is worth noting that there may be an area instead of a mere piece of line where in any place you are equidistant to the two national coasts (one area at most: the Voronoï cell, if any, originated by the joint point of the two coasts).

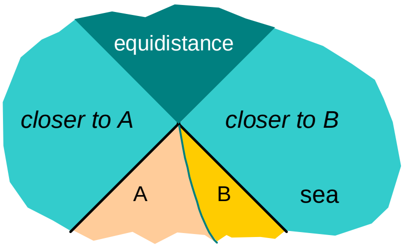


Figure 3.2 Sea area equidistant to two states

This happens when the land boundary ends on a convex part of the geographical coast, ie. on a protruding (however small) of the land into the ocean (Figure 3.2). Because, in this area, distance from sea to coast amounts to distance from sea to the last land boundary marker, any point there (and thus any possible line within, straight, bordering, spiraling, wriggling, crisscrossing or even moving!) is geometrically equidistant to the two states. The Handbook on the Delimitation of Maritime Boundaries (Doals 2000), in its otherwise impressively scrupulous review of possible configurations and cases, does not address the issue of equidistant areas. By using solely the expression "equidistant line", it may suggest that only lines come into play when geometrical equidistance is looked for. Such "cape-configurations" may be rare in actual land borders between states, and boundaries are culturally linear nowadays (Foucher 1994:39); yet the omission may increase the natural difficulty of apprehending or visualizing the geometry of equidistant borders in general. This very difficulty, it must be stressed, is resolved by Voronoï diagrams inasmuch as 1) they take the full geometry of the baselines into account for the computation, and, 2) when displayed they show, pedagogically, the influence and effects of every detail in the shapes of the baselines.

The algorithm for computing a maritime border is thus most simple, when using Voronoï diagrams:

- Construct the Voronoï diagram on the *n* elements (points and segments) of the baselines, storing also, as a pair of attributes for each edge between two elements, the nature of the coast of either element,
- Select edges of the diagrams with different natures (Figure 3.3).

If equidistant areas are felt undesirable, the algorithm may continue into the identification of the possible equidistant area, and stop to ask for man-made completion of the "median" line, or adopt some predefined routine (eg. straight line from land-node to sea-node, or best spline prolongation of land border from coast to off-shore median line).
The algorithm could also "deflate", so to speak, this equidistant "bubble", again using Voronoï. The contour of an equidistant area may be said indeed to be composed of two parts:

the line in the sea where, under the influence of the last land boundary marker, the points are no longer strictly closer to coast A, and the line where the points are no longer strictly closer to coast B. Were these two lines considered as inheriting the nationalities of coast A and of coast B respectively, the equidistant line in-between could also be looked for voronoïcally. The case is comparable to the banks of a lake between two countries, yet with the interesting properties that these banks are both mutually "concave", in geographical terminology, and, in the plane, composed of straight segments and of pieces of parabolas that have the same focus. The result is a line indeed (parabola–parabola equidistance in the plane resulting, however, in higher-order polynomial curves), which can be proved (but won't be detailed here) from the observation that the original contour is geometrically convex (the two "banks" being geographically concave).

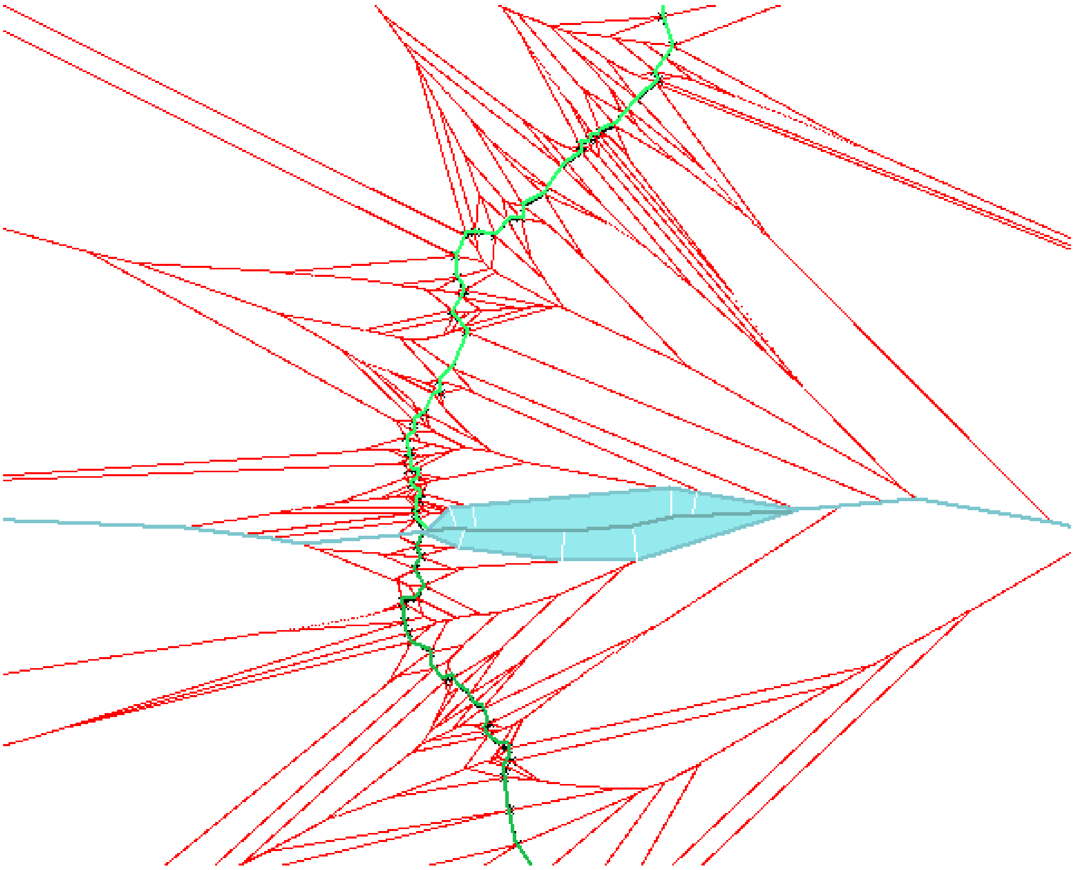


Figure 3.3 Two adjacent coasts; the Voronoï diagram of their elements; equidistance into the sea: westwards (land supposed on the right) a simple median line, eastwards (land supposed on the left) the equidistance bubble on the coast and its Voronoï deflation.

3.2.3 Example of the Qatar – Saudi Arabia Border

The Voronoï principles above helped the French national mapping agency (Institut Géographique National) delineate the maritime border between Saudi Arabia and Qatar in Salwa Bay in 1999. Detailed accounts, on both the technical and the epistemological levels, can be found in Bertolotto-Leidinger & Hangouët, 2003. The two countries wanted the median line for border, choosing for baselines the low-tide coastlines, as captured by IGN for the cartography of the land boundary at a 1:30,000 scale. IGN survey engineer Pascal Boulerie, who knew the requirements of the countries, the power of Voronoï diagrams, and the works carried on Voronoï diagrams at IGN's COGIT laboratory by JF. Hangouët, brought Research and Production together to collaborate on the delineation of the maritime border. Some 16.000 coastal points were used to compute the median line, which came out (Figure 3.4) in a fraction of a minute.

Figure 3.4 Median Line in Salwa Bay

The median line was accepted by the two states as geometrically valid, yet the resulting 250 points and more were judged intractable for cartographic and legal purposes. They agreed on a simplified line which compensates water areas. This solution is quite usual, as addressed in DOALS, 2000:49; it can be illustrated also by the Convention between Switzerland and France on their common border in Lake Geneva:

The border demarcation in Lake Geneva is composed of a median line (and…). The median line is defined, theoretically, as the locus of the centers of the circles inscribed between the Swiss and French banks.
The theoretical line comes to be replaced, however, and for practical reasons, by a polygonal line of six segments which achieves compensation of surface areas.
[SoEG translated from the French by the authors]

The final borderline, it may be supposed, could never have been traced nor even accepted, but for the geometrically exact and unbiased median line.

3.3 Maritime Zones

3.3.1 General points

Different maritime zones are identified by the 1982 Convention on the Law of the Sea (Doals, 2000:8): territorial sea (the breadth of which cannot extend beyond 12 nautical miles from the baseline), contiguous zone (breadth limited to 24 nm), exclusive economic zone (breadth limited to 200 nm), continental shelf (breadth limited to 200 nm from the baseline or in special cases to 350 nm from baseline and 100 nm from the 2,500-metre isobath) (Doals, 2000:10).

3.3.2 Voronoï Diagrams for Maritime Zones

Maritime zones (territorial sea, contiguous zone, exclusive economic zone, continental shelf) can be seen as what GIS designers are used to calling "d-wide buffers" off the coasts (eg. d=12, d=24, d=200, d=350 nautical miles) or off the 2,500-metre isobath for the special cases of continental shelves (d=100 nm). In common giss, such buffers are usually computed through implicit rasterization, even on vector data. Voronoï diagrams however, when stored with useful attributes (notably the minimum and maximum distances from each edge to its originating element, as documented in Hangouët, 2000), make vector computation of planar "d-wide buffers" straightforward. Since Voronoï cells make the computation of distance to the original polyline amount to the computation, within each cell, of the distance to the point or segment of the polyline which originates the cell, simply:

- select Voronoï edges with dmin < d and dmax > d and their associated cells,
- in each cell,
  if originated by a point, draw an arc of circle, centered on the point, with radius d,
  if originated by a segment, draw a parallel segment at distance d,
  stop at intersections with Voronoï edges.

The result on the map is illustrated on Figure 3.5 below.
On the sphere, equivalently, buffers are limited by circle arcs "centered" on coastal points or parallel to coastal great circle segments.

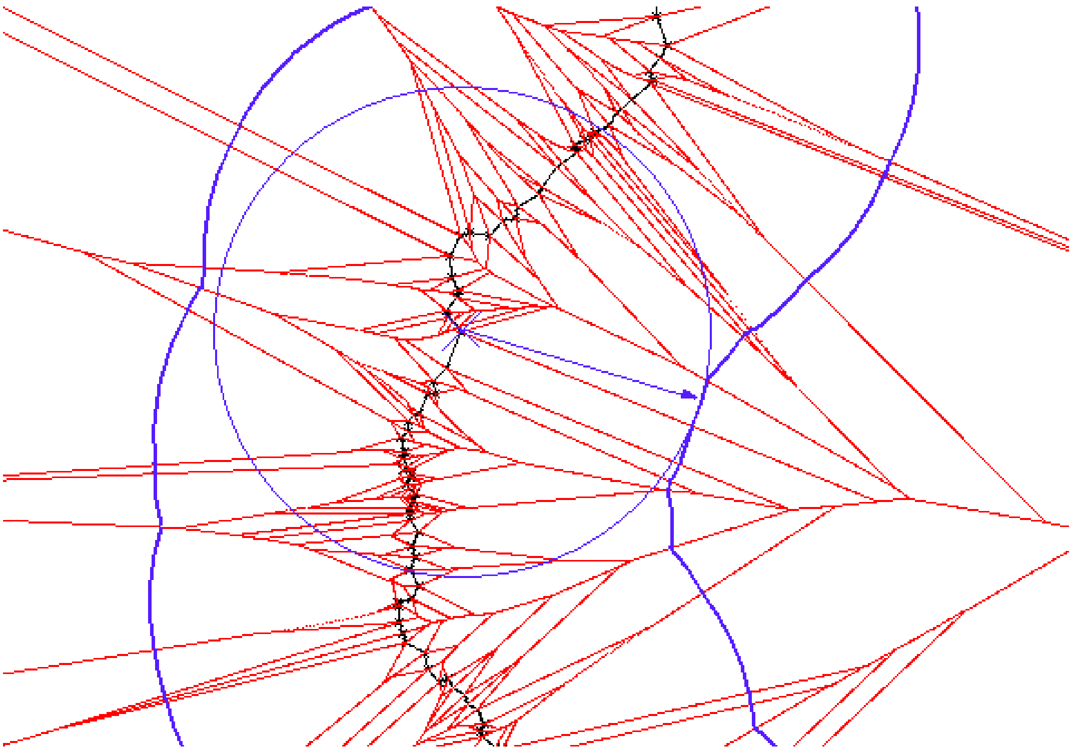


Figure 3.5 A geographic line, the Voronoï diagram on its point and segment elements, two buffers (here of a same breadth) eastwards and westwards from the line.

3.3.3 Voronoï Diagrams and Christensen's Automated Waterlining

Buffers of regularly (or increasingly) increasing breadths, accumulated from the bank and off, result in what cartography calls waterlining, a beautiful ornamental technique apparently designed by map engravers and performed for an approximate two hundred years (from the middle of the XVIIIth century to the middle of the XXth century, according to evidence found in IGN's most accessible map collections). Yet "such lines, equidistant from the shores, do not stem from mere cartographic amusement. They are used intensively to establish national and legal borders in waters, and thus prove of high practical, political and legal significance" (Imhof 1961:69, translated by the authors): Where, actually, two simultaneous trains of increasing buffers meet (and indeed, at each salient point, equidistant to two elements, thus a Voronoï point), the median line appears geometrically (see Figure 3.6).

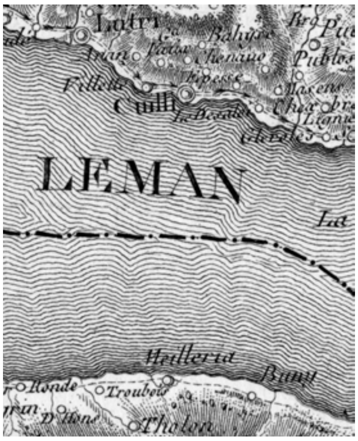


Figure 3.6 Waterlining, median line between Switzerland and France, and latent Voronoï on Lake Geneva (extract from sheet 6 of Carte du Théâtre de la Guerre d'Italie, by Bacler d'Albe, 1801, Collection Cartothèque de l'Institut Géographique National)

This is the very property of increasing buffers that A. Christensen, most pertinently (and in the great cartographic tradition: both most precisely and most beautifully), uses to compute maritime borders (Christensen 1999, 2002). Christensen actually does not compute Voronoï, but buffers directly, and their intersections. The method he uses is somewhat time-consuming as, he writes, "the waterlining step (…) is proportional to the square of the product of the number of waterlines and the average number of vertices in a waterline" (Christensen 2002:6/10). With the geometrically equivalent Voronoï method, computation times are reduced, since the algorithmic complexity is O(n . log n), when n elements (n points or, equivalently for algorithmic complexity, n segments) compose the initial coastlines. This purely computational aspect however should not be counted as an advantage of the Voronoï method, but only as one of the signs that Voronoï diagrams lend themselves easily and tractably to automated handling of geographical data (see Hangouët 2000, for an argumentation and a list of various basic automated spatial operations made easy with Voronoï diagrams).

3.3.4 Voronoï Diagrams for the Determination of Baselines

In the preceding sections, the baseline, from which seaward distances are computed, was considered as input for the Voronoï computation of median line and boundary line. Yet Voronoï diagrams may prove just as helpful and handy also for the determination of the baseline itself from the full coastline, if needs be. When "the coastline is deeply indented and cut into, or if there is a fringe of islands along the coast in its immediate vicinity" (1982 Convention on the Law of the Sea, quoted in DOALS 2000:4), it has to be "generalized" (in the cartographic sense) in places to become "straight". According to Doals 2000:6, indeed: "The straight baselines must be drawn to satisfy several requirements: they must not depart from the general direction of the coast, the sea areas lying within the lines must be sufficiently closely linked to the land domain to be subject to the regime of internal waters, they shall not be drawn to and from low-tide elevations, and they shall not cut off the territorial sea of another State form the high seas or an exclusive economic zone." It may be added that water is lost in subsequent boundary determinations if land comes to be cut off at this stage: most line generalization algorithms, including the most widespread "Douglas & Peucker" algorithm (Douglas & Peucker, 1973), will fail in this. The operation that mathematical morphology calls "full r-wide closure", however, will not.

This "full r-wide closure" can be seen as the result of a r-wide buffer off the coastline (including, possibly, the contour of nearby islands), the limit of which is r-wide buffered back landward (proximate islands being possibly amalgamated to land in the process). The output line follows the general direction of the coast, but not the smaller indentations (the larger the value of r, the fewer the indentations left. Infinite r results in the so-called "convex hull" of the line).

The result is not "straight" exactly, being composed of line segments and circular arcs (in the plane), but since it can be computed quickly, and needs only one input parameter, it allows for on-the-fly and simple simulations and demos at bilateral or multilateral negotiation stages. Also, using for r the value used for the determination of whatever maritime zone from a coastline reveals the baseline with which the original coastline is equivalent, as seen from the boundary at sea.

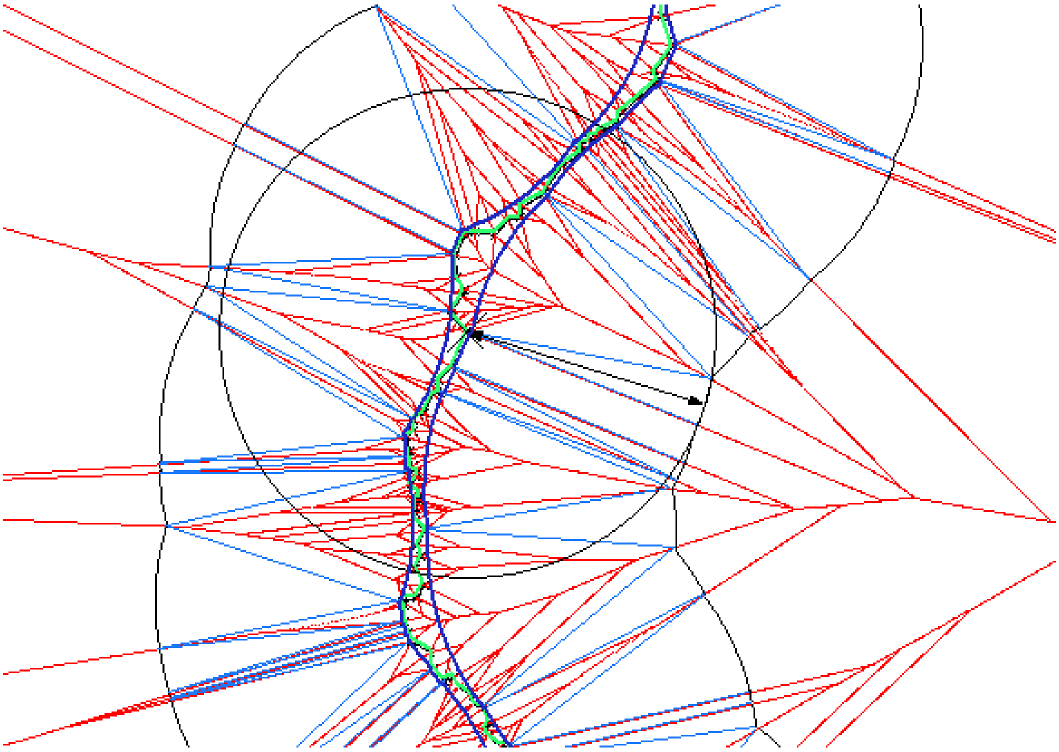


Figure 3.7 Coastline; Voronoï diagram on its elements; full r-wide closure westwards; full r-wide closure eastwards

The computation of the full r-wide closure in the plane with Voronoï runs as follows:

- select Voronoï edges with dmin<r and dmax > r and the points Pr on edges at distance r,

- project each Pr on the coastal points and segments from which it is equidistant,
- join the projections, either by a segment when they are on a same coastal segment, or, when they aren't, by a circular arc, of radius r, centered on their common Pr. (Figure 3.7)

4. CONCLUSION

The GPS real time kinematics technique was tested by IGN for the setting up of the main and particularly the intermediate markers of the Saudi Arabia-Qatar border. This technique has shown its efficiency in term of speediness as well as in term of accuracy. The comparisons and statistics computed after a global GPS re-observation of all the border monuments gave 5 mm for the standard deviation of the discrepancy between observed and computed positions on the theoretical line (maximum discrepancy: 15 mm).
The paper has also made apparent that maritime median lines, maritime zones, cartographic waterlining, straight baselines, can be computed most simply and most naturally when Voronoï diagrams are constructed from the coastlines. The illustrations given in the paper will show that, in addition, Voronoï diagrams provide both expressive and precise visualization aids to explain apparent geometrical entanglements and to understand possible configurations (such as the equidistant bubble that may appear offshore, unmentioned in the references consulted). This, we argue, is of particular importance when legal boundaries are at stake and investigative negotiations must be carried out. Confident in the powerfulness of Voronoï diagrams, and importing expertise on these structures from IGN's Research Department, IGN's Geodesy and Leveling Department has decided to include the computation of Voronoï in its geodetic computation system, for purely geometric delineation purposes, yet also with a view to simulation, negotiation and pedagogical applications.

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BIOGRAPHICAL NOTES

Gérard Cosquer, survey engineer, has been working at the French "Institut Géographique National" for some 28 years and notably, in the last 12 years, as project manager for the establishment of the Yemeni National Geodetic and Leveling Network, for the Saudi Arabia-Qatar border demarcation and, more recently, as head of the Geodetic Networks Section in the Geodesy and Leveling Department.

Jean-François Hangouët, survey engineer, PhD, has been working at the French Institut Géographique National for some 10 years, both as a researcher on cartographic generalization and spatial analysis and, more recently, as a member of IGN’s Quality Management unit.

CONTACTS

Gérard Cosquer
Institut Géographique National,
Service de la Géodésie et du Nivellement
2-4 avenue Pasteur
F-94165 Saint-Mandé cedex
FRANCE
Tel. +33 1 43 98 81 69
Fax +33 1 43 98 84 50
Email: gerard.cosquer@ign.fr
www.ign.fr

Jean-François Hangouët
Institut Géographique National,
Mission Qualité
2-4 avenue Pasteur
F-94165 Saint-Mandé cedex
FRANCE
Tel. + 33 1 43 98 80 26
Fax + 33 1 43 98 81 71
Email: jean-francois.hangouet@ign.fr
www.ign.fr